ME 406 - DYNAMICAL SYSTEMS
SPRING SEMESTER 2009

INSTRUCTOR

Alfred Clark, Jr., Hopeman 329, x54078; E-mail: clark@me.rochester.edu
Office Hours: M T W Th F 1600 – 1800.

COURSE TIME AND PLACE

T Th 1400 – 1515 Hylan 306

PREREQUISITES

The prerequisites are linear algebra (particularly the eigenvalue problem) and basic ordinary differential equations, including matrix methods for systems of equations. Because the linear algebra is so important, we will spend some time reviewing it at appropriate points in the course. In addition to linear algebra and ordinary differential equations, some prior experience with more advanced applied mathematics would be helpful -- for example, a course such as ME201/MTH 281. It would also be helpful (but not essential) to have some knowledge of elementary vibration theory. If you have any concerns about your background, I would be happy to discuss them with you.

COURSE DESCRIPTION

This course is an applied mathematics course designed to be an introduction to dynamical systems. Dynamical systems is a very large subject which includes such topics as nonlinear differential equations, iterated mappings, and fractals. Our course will concentrate on the nonlinear differential equations. The purpose of the course is to study the following question: What are the possible events in a system governed by nonlinear ordinary differential equations? The coverage will be more by example than by theorem. A major part of the course will be hands-on experience in exploring equations with the computer. The hardware and software to be used is described in more detail below.

ASSIGNMENTS

There will be 11 written homework assignments, most of which will involve computer work. There will be a three-hour mid-term exam covering the first half of the course on Wednesday April 1 (time to be arranged). The final exam, with emphasis on the second half of the course, is on Friday May 8, 1600 – 1900. The grading weights are homework 50%, mid-term 25%, and final exam 25%.

OUTLINE OF COURSE

The outline below is subject to changes as we proceed through the course. In addition to the topics listed, there will be applications at appropriate points in the course. On the last page of this handout, there is a detailed schedule of classes. We may deviate from this schedule as we go through the semester. An updated schedule will always be available online.

1. Plane Autonomous Systems
Phase plane plots; stability of equilibrium for linear systems; stability for nonlinear systems by linearization; Liapunov's method for stability; periodic solutions; stability of periodic solutions; global phase portraits; bifurcations.

2. Higher Order Autonomous Systems
Review of linear algebra and matrix methods for linear systems; local behavior near equilibrium points; Floquet theory and stability of periodic solutions; volume in phase space; general features of Lorenz equations; transition to chaos for Lorenz equations; fully developed chaos for Lorenz equations; tent map and the Lorenz equations; Liapunov exponents.

3. Selected Additional Topics (if time permits)
The driven pendulum; dynamics of infectious diseases; the logistic map.

TEXTBOOK

The required text is **Nonlinear Dynamics and Chaos**, Steven H. Strogatz, Addison-Wesley, 1994, paperback edition 2001. Although we will cover about two-thirds of the material in the book, we will not follow it in detail in class lectures. It is an excellent text – practical, authoritative and very readable.

REFERENCES

The literature in this field is extensive and varied in difficulty. The books on the short list below have been selected to be especially useful for broad coverage at the basic level of ME 406. The ones with an (R) designation at the end of the reference provide essential background for our course and are on reserve in Carlson. More specific references on special topics will be given during the course.

Popular Treatments of Dynamical Systems and Chaos

Does God Play Dice?, I. Stewart, 2nd edition,Blackwell, 2002. A superb overview of the field by a mathematician who has worked in the area. (R)

Chaos, J. Gleick, Viking, 1987, available in paperback. Less technical than Stewart's book, but still providing a very nice overview.

The Essence of Chaos, E.N. Lorenz, University of Washington Press, 1993. Very interesting discussions of chaos, including some history of Lorenz's own discoveries.

Background in Differential Equations

Differential Equations, P. Blanchard, R.L. Devaney, and G.R. Hall, Brooks/Cole 1998. This outstanding text has been developed for a first course in differential equations, approaching the subject via systems of equations. It has very clear and basic discussions of many of the topics we cover, at a more elementary level than our course. (R)

Ordinary Differential Equations, G. Birkhoff and G.-C. Rota, 4th edition, Wiley 1989. This is an intermediate and excellent general text on ordinary differential equations. Chapter 5 is a good introduction to plane autonomous systems. (R)

Theory of Ordinary Differential Equations, E. A. Coddington and N. Levinson, McGraw-Hill, 1955, Krieger reprint, 1984. Very advanced and comprehensive treatment, including a number of topics of importance in dynamical systems theory.

General Treatments of Nonlinear Differential Equations and Dynamical Systems

Differential Equations, Dynamical Systems, and an Introduction to Chaos, 2nd edition, M.W. Hirsch and S. Smale, and R.L. Devaney, Elsevier, 2005. This is a new edition of the elegant 1974 text by Hirsch and Smale. The most useful references for our course are our text, this book, and the books below by Jordan and Smith and Meiss. (R)

Nonlinear Ordinary Differential Equations: An Introduction for Scientists and Engineers, D.W. Jordan and P. Smith, 4th edition, Oxford Press, 2007. This book is very readable, has broad coverage, and many excellent examples and problems. (R)

Differential Dynamical Systems, James Meiss, SIAM, 2007. This recent applied math book was one of the possible texts for our course. You will find very helpful treatments in here of most of the topics covered in our course (and much more). (R)

Differential Equations and Dynamical Systems, L. Perko, Springer-Verlag, 3rd edition, 2001. This excellent book is written for mathematicians, but the background required is basic, and the book is not, for our purposes, overly abstract. If you want a treatment which is mathematically precise but accessible, this is the book to read. Another strength of the book is its organization, and the fact that it focuses completely on the fundamentals. The four chapters in the book are (1) Linear Systems, (2) Nonlinear Systems: Local Theory, (3) Nonlinear Systems: Global Theory, and (4) Nonlinear Systems: Bifurcation Theory. (R)

Nonlinear Differential Equations and Dynamical Systems, F. Verhulst, Springer-Verlag, 2nd edition, 1996, paperback. This book has more of a pure math flavor then Jordan and Smith, but it is still at an introductory level. It has less on chaos than we will cover. It is concise but very readable and has many examples. (R)