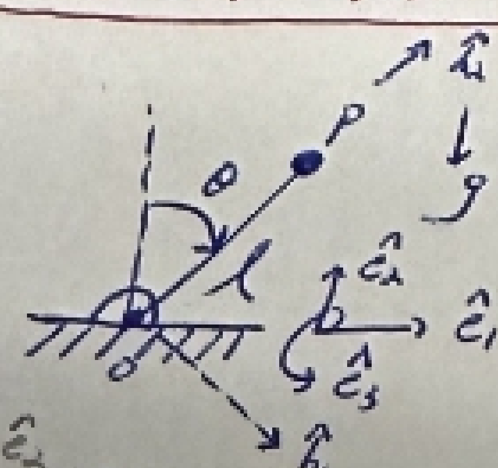
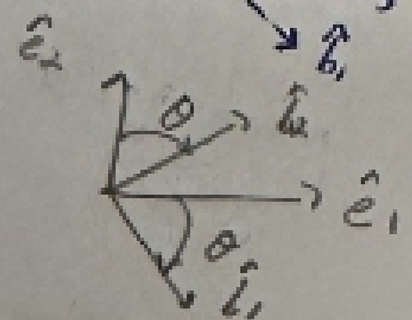
AAE 340 FA 2017 E 2 problem 2 | INVERTED PENDULUM





(a) ${}^{c}H^{o} = m\bar{R}^{op} \times {}^{i}\bar{v}^{op}$; $\bar{R}^{op} = l\hat{b}_2$; ${}^{i}\bar{v}^{op} = ({}^{i}\bar{\omega}^{b} \times l\hat{b}_2)$

${}^{i}\bar{v}^{op} = (-\dot{\theta}\hat{b}_3 \times l\hat{b}_2) = \dot{\theta} l \hat{b}_1$

${}^{c}H^{o} = ml\hat{b}_2 \times \dot{\theta} l \hat{b}_1 = -ml^2\dot{\theta}\hat{b}_3$

$\bar{M}^{o} = \bar{R}^{op} \times F_p = (l\hat{b}_2 \times (-F_c\hat{b}_2 - mg\hat{e}_2))$

$M^{o} = l\hat{b}_2 \times [-mg(\cos\theta\hat{b}_2 - \sin\theta\hat{b}_1)]$

$M^{o} = l\hat{b}_2 \times (-mg\cos\theta\hat{b}_2 + mg\sin\theta\hat{b}_1)$

$M^{o} = -\cancel{mg}\cancel{l}\sin\theta\hat{b}_3 = -\cancel{m}l\cancel{l}\ddot{\theta}\hat{b}_3$

$g\sin\theta = \ddot{\theta} l$: $\boxed{\ddot{\theta} - \frac{g}{l}\sin\theta = 0}$ EOM

(b) $\sin\theta \approx \theta$, $\boxed{\ddot{\theta} - \frac{g}{l}\theta = 0}$

(c) $r^2 - \frac{g}{l} = 0$; $r = \pm\sqrt{\frac{g}{l}}$

$\theta(t) = c_1 e^{\sqrt{\frac{g}{l}}t} + c_2 e^{-\sqrt{\frac{g}{l}}t}$: $\theta(0) = \theta_0$. $\dot{\theta}(0) = \dot{\theta}_0$

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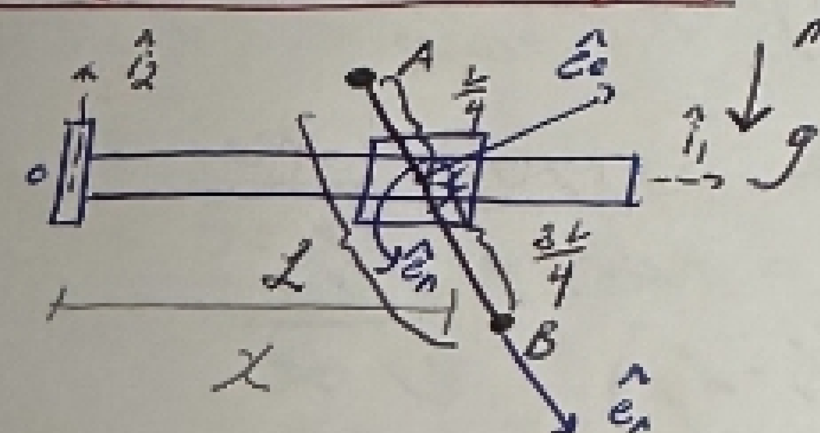
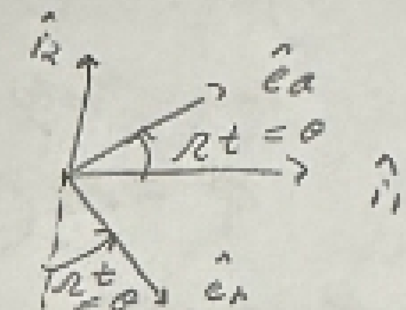
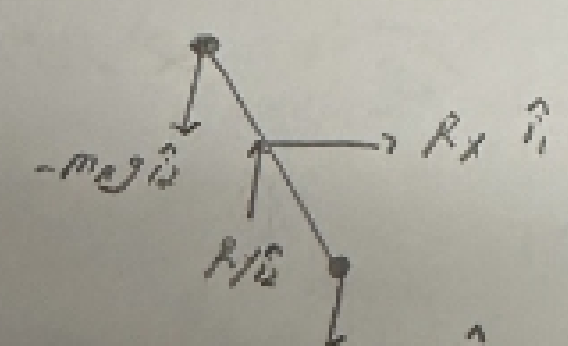
(1)



$m = m_A = m_B$, rod rotates about C with Ω (not constant). Let $\Omega = \dot{\theta}$



(a) FBD :



(b) ${}^{i}H^{o}$; ${}^{i}H^{o} = m\bar{R}^{oA} \times {}^{i}\bar{v}^{oA} + m\bar{R}^{oB} \times {}^{i}\bar{v}^{oB}$

c.o.m. : $\bar{R}^{oo} = \dfrac{m(x\hat{i}_1 - \frac{L}{4}\hat{e}_r) + m(x\hat{i}_1 + \frac{3L}{4}\hat{e}_r)}{2m}$

$\bar{R}^{oo} = \dfrac{2mx\hat{i}_1}{2m} + \dfrac{\frac{2L}{4}\hat{e}_r}{2} = x\hat{i}_1 + \frac{L}{4}\hat{e}_r$

$\bar{R}^{oA} = \bar{R}^{oo} + \bar{R}^{oA}$; $\bar{R}^{oA} = x\hat{i}_1 - \frac{L}{4}\hat{e}_r - x\hat{i}_1 - \frac{L}{4}\hat{e}_r = -\frac{L}{2}\hat{e}_r = \bar{R}^{oA}$

$\bar{R}^{oB} = \bar{R}^{oo} + \bar{R}^{oB}$; $\bar{R}^{oB} = x\hat{i}_1 + \frac{3L}{4}\hat{e}_r - x\hat{i}_1 - \frac{L}{4}\hat{e}_r = \frac{L}{2}\hat{e}_r = \bar{R}^{oB}$

${}^{i}H^{o} = \frac{-mL}{2}\hat{e}_r \times -\frac{L\dot{\theta}}{2}\hat{e}_\theta + \frac{mL}{2}\hat{e}_r \times \frac{L\dot{\theta}}{2}\hat{e}_\theta = \frac{mL^2\dot{\theta}}{\cancel{4}2}\cdot\cancel{2} = \frac{mL^2\dot{\theta}}{2}\hat{e}_n = {}^{i}H^{o}$

$\boxed{{}^{i}H^{o} = \frac{mL^2\Omega}{2}\hat{e}_n}$

(c) $P_o(t) = \int_{t_1}^{t_2} \bar{F}^{(ext)}\,dt$ sum of external forces must be 0 for linear momentum to be conserved. Thus, linear momentum is only conserved in the $\hat{e}_n = \hat{i}_3$ direction.

(d) c.o.m. if block C can move (massless),

$\bar{R}^{oo} = \dfrac{m(x\hat{i}_1 - \frac{L}{4}\hat{e}_r) + m(x\hat{i}_1 + \frac{3L}{4}\hat{e}_r)}{2m} = x\hat{i}_1 + \frac{L}{4}\hat{e}_r$

$= x\hat{i}_1 + \frac{L}{4}(-\cos\theta\hat{i}_2 + \sin\theta\hat{i}_1) = (x + \frac{L}{4}\sin\theta)\hat{i}_1 - \frac{L}{4}\cos\theta\hat{i}_2$

$\boxed{\bar{R}^{oo} = (x + \frac{L}{4}\sin(\Omega t))\hat{i}_1 - \frac{L}{4}\cos(\Omega t)\hat{i}_2}$