Mechanics

Physics 151

Lecture 13

Oscillations

(Chapter 6)

What We Did Last Time

- Studied oscillation
  - Discussed general features of multi-dimensional oscillators
- Equation of motion → Eigenvalue problem $\mathbf{Va} = \omega^2 \mathbf{Ta}$
  - Showed that oscillating solutions exist $\eta_i = C a_i e^{-i\omega t}$
  - Eigenvalues ω^2 are positive definite
    - Provided that V is minimum at the equilibrium
- Principal axis transformation diagonalizes **T** and **V**
  - Normal coordinates behave as independent oscillators

Forced Oscillation

- Force F_j is applied to coordinate $q_j = q_{0j} + \eta_j$
  - Linear force if q_j is a Cartesian coordinate
- Switch to normal coordinates $\{\zeta_i\}$ ← $\eta_j = A_{ji}\zeta_i$
  - New generalized force is $Q_i = F_j \dfrac{\partial \eta_j}{\partial \zeta_i} = F_j A_{ji}$ or $\mathbf{Q} = \tilde{\mathbf{A}}\mathbf{F}$
- Without force, ζ_i was satisfying $\ddot{\zeta}_i + \omega_i^2 \zeta_i = 0$
  - Now it must satisfy $\ddot{\zeta}_i + \omega_i^2 \zeta_i = Q_i$

No cross-terms between coordinates

Sinusoidal Force

- Suppose F_j is sinusoidal with time $F_j(t) = \mathrm{Re}\left(F_j e^{-i\omega t}\right)$ — Complex number
  - Pressure from sound wave
  - Microwave radiation on molecule in food
  - Any periodic force will do → Fourier series
- Generalized force on a normal coordinate is

  $Q_i(t) = Q_i e^{-i\omega t} = F_j A_{ji} e^{-i\omega t}$
  - Equation of motion is

    $\ddot{\zeta}_i + \omega_i^2 \zeta_i = Q_i e^{-i\omega t}$ That's easy to solve

General Solution

$$\ddot{\zeta}_i + \omega_i^2 \zeta_i = Q_i e^{-i\omega t}$$

- General solution for inhomogeneous equation is

1 particular solution + any solution for the homogeneous equation

  - In this case $\zeta_i = B_i e^{-i\omega t} + C_i e^{-i\omega_i t}$ with $B_i = \dfrac{Q_i}{\omega_i^2 - \omega^2}$

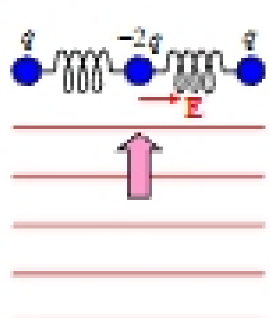
    Forced solution / Force-free solution
  - C_i is arbitrary → Fixed by the initial conditions
  - Amplitude B_i of forced oscillation depends on $\omega_i^2 - \omega^2$
    - The closer to the natural frequency, the bigger

An example will help here

Triatomic Molecule

- Take the triatomic molecule from last week
  - Suppose the atoms are charged
    - Molecule is polarized
  - Plane EM waves are arriving
    - $E = E_0 e^{-i\omega t}$ at the atoms
  - Forces are

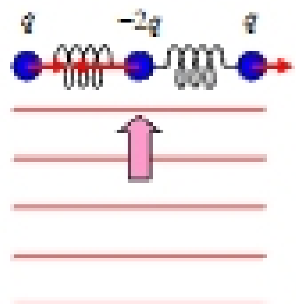
    $$\mathbf{F} = \begin{bmatrix} q \\ -2q \\ q \end{bmatrix} E_0 e^{-i\omega t}$$
  - Switch to the normal coordinate
    - Use $\mathbf{A}$ from last week



Triatomic Molecule

$$\mathbf{F} = \begin{bmatrix} q \\ -2q \\ q \end{bmatrix} E_0 e^{-i\omega t}$$

$$\mathbf{A} = \frac{1}{\sqrt{6m}} \begin{bmatrix} \sqrt{2} & \sqrt{3} & 1 \\ \sqrt{2} & 0 & -2 \\ \sqrt{2} & -\sqrt{3} & 1 \end{bmatrix} \qquad \boldsymbol{\eta} = \mathbf{A}\boldsymbol{\zeta}$$



- Generalized force for ζ is

$$\mathbf{Q} = \tilde{\mathbf{A}}\mathbf{F} = \begin{bmatrix} 0 \\ 0 \\ \sqrt{6/m} \end{bmatrix} qE_0 e^{-i\omega t}$$

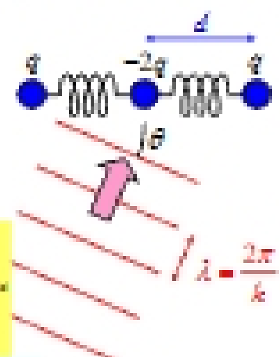
- Only ζ_3 gets forced oscillation
  - Direction of the forces are just right for this mode

Triatomic Molecule

$$\mathbf{A} = \frac{1}{\sqrt{6m}} \begin{bmatrix} \sqrt{2} & \sqrt{3} & 1 \\ \sqrt{2} & 0 & -2 \\ \sqrt{2} & -\sqrt{3} & 1 \end{bmatrix}$$

- Send the EM waves at an angle
  - F_j have different phases now

$$\mathbf{F} = \begin{bmatrix} qe^{-i(\omega t + dk\sin\theta)} \\ -2qe^{-i\omega t} \\ qe^{-i(\omega t - dk\sin\theta)} \end{bmatrix} E_0 \cos\theta$$



A bit of work

$$\mathbf{Q} = \begin{bmatrix} \frac{2}{\sqrt{3m}}(\cos\delta - 1) \\ -i\sqrt{\frac{2}{m}}\sin\delta \\ \sqrt{\frac{2}{3m}}(\cos\delta + 2) \end{bmatrix} qE_0 \cos\theta e^{-i\omega t}$$

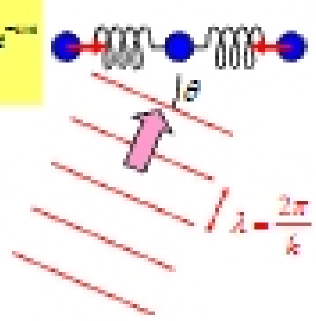
$\delta \equiv dk\sin\theta$

Triatomic Molecule

$$\mathbf{Q} = \begin{bmatrix} \frac{2}{\sqrt{3m}}(\cos\delta - 1) \\ -i\sqrt{\frac{2}{m}}\sin\delta \\ \sqrt{\frac{2}{3m}}(\cos\delta + 2) \end{bmatrix} qE_0 \cos\theta e^{-i\omega t}$$

- Oscillation of ζ_2 is maximized by

$$\delta \equiv dk\sin\theta = \frac{\pi}{2} \Rightarrow \mathbf{F} = \begin{bmatrix} -i \\ -2 \\ +i \end{bmatrix} qE_0 \cos\theta e^{-i\omega t}$$



  - 1st and 3rd atoms are driven in the opposite directions

Generally, a normal mode is excited when the external forces push the particles in the direction the mode "wants to" move them