PROBLEMS

9.1. A long thin steel cable of unstressed length l is hanging from a fixed support, as illustrated in Fig. 9P.1. Assume that the origin of coordinates is at the support and that x measures positive as shown. Assume that the steel cable has the following constants:

Cross-sectional area	$A = 10^{-4}$ m^2
Young's modulus	$E = 2.0 \times 10^{11}$ N/m^2
Mass density	$\rho = 7.8 \times 10^3$ kg/m^3
Maximum allowable stress	$T_{\max} = 2 \times 10^9$ N/m^2

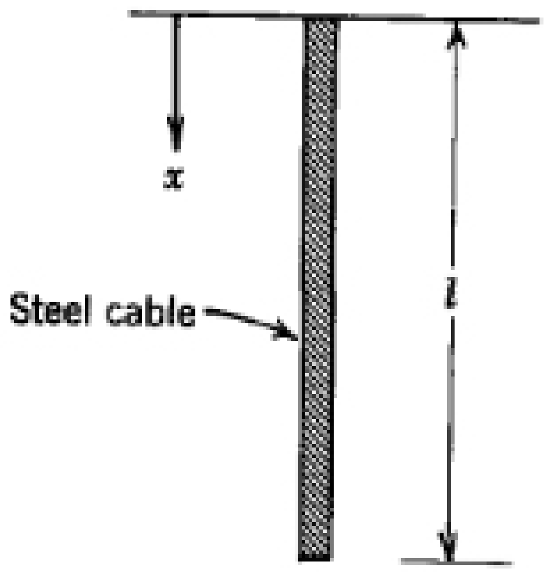

Fig. 9P.1

(a) Find the length of cable l for which the maximum stress in the cable just equals the maximum allowable stress.
(b) Find the displacement δ and stress T in the cable as functions of x.
(c) Find the total elongation of the cable.

9.2. Two thin elastic rods are arranged as shown in Fig. 9P.2. The first rod has modulus of elasticity E_1, density ρ_1, and cross-sectional area A_1. It is attached at one end to a rigid wall and at the other to a very thin rigid plate of mass m and area A_m. On the other side of

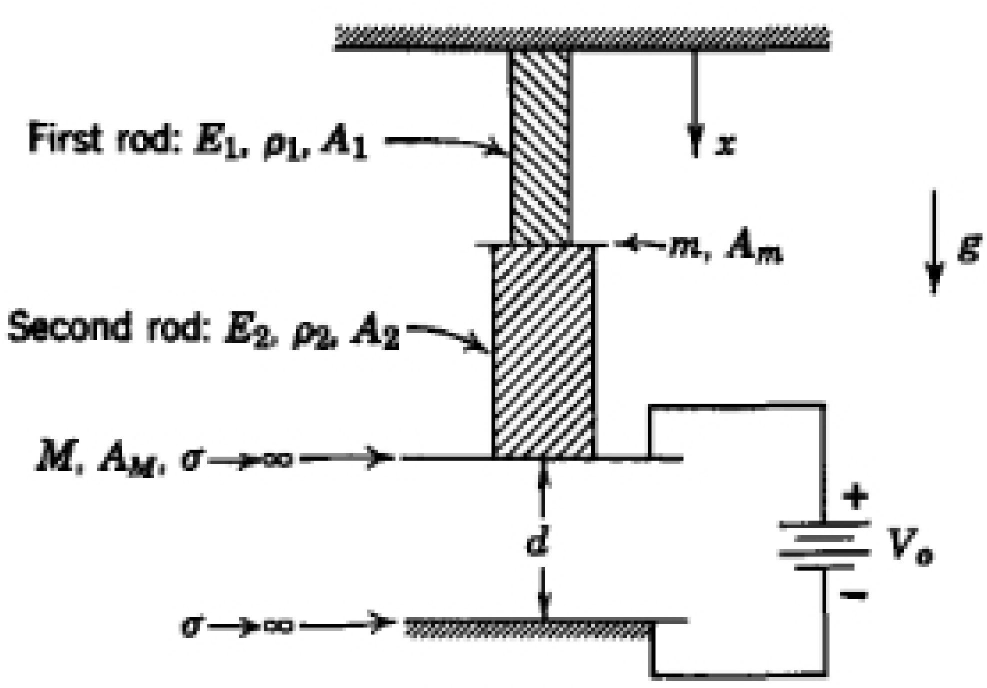

Fig. 9P.2

this plate is attached a second thin elastic rod with elastic modulus E_2, density ρ_2, and cross-sectional area A_2. The other end of the second rod is fixed to a perfectly conducting thin plate with mass M and area A_M. This plate is held at a potential V_0 with respect to a second capacitor plate a distance d away. In the absence of gravity and with $V_0 = 0$, the length of the first rod is L_1 and the length of the second is L_2. Assuming now that the system is immersed in a gravitational field g and that $V_0 \neq 0$, find the following:

(a) The stress in the first rod $T^{(1)}(x)$ and the displacement in the first rod $\delta^{(1)}(x)$.
(b) The stress in the second rod $T^{(2)}(x)$ and the displacement in the second rod $\delta^{(2)}(x)$.

9.3. In Fig. 9P.3 a thin elastic rod of cross-sectional area A, equilibrium length l, elastic modulus E, and mass density ρ is fixed at one end ($x = 0$) and attached to a rigid mass M

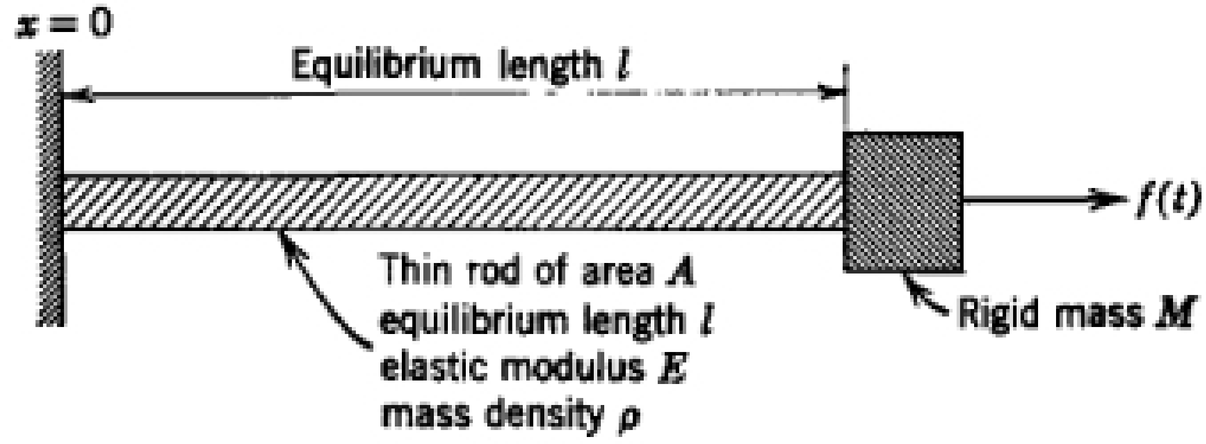

Fig. 9P.3

at the other ($x = l$). The mass is driven by a force source $f(t)$. The system constants are such that the mass M is much greater than the mass of the elastic rod; that is,

$$M \gg \rho A l.$$

The force source is constrained to be

$$f(t) = \mathrm{Re}\,(f_0 e^{j\omega t}),$$

where f_0 and ω are positive real constants. The system is operating in the sinusoidal steady state. Neglect gravity.

(a) Find the displacement $\delta(x, t)$ and stress $T(x, t)$ in the elastic rod.
(b) Show a lumped-parameter mechanical system that represents the behavior of the system in Fig. 9P.3 for low frequencies (from $\omega = 0$ up to and including the lowest resonance frequency). Evaluate the equivalent elements in terms of the given parameters.

9.4. A long thin elastic rod with cross-sectional area A, unstressed length l, modulus of elasticity E, and mass density ρ is constrained at one end by the three ideal, lumped elements

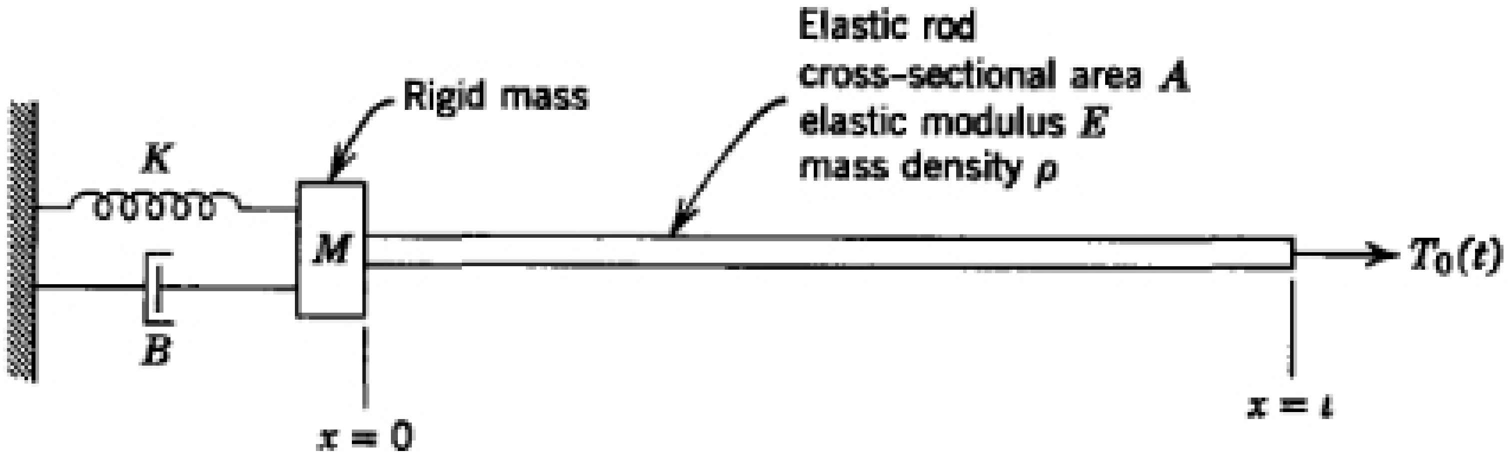

Fig. 9P.4