251y0234 12/02/02 **ECO251 QBA1** _

THIRD EXAM

NOVEMBER 20, 2001 **Name: _____Key__________**

(Open this document in 'Print Layout' view!) **Section Enrolled:** (Circle) MWF 10 MWF 11 TR 11

Since there is no correct answer for the Take-home exam a possible solution appears below. Many errors you made are in 'Things that you should never do on a Statistics Exam or Anyplace Else.' Wake up and read it!

251x023a 11/16/01 **ECO251 QBA1** _

THIRD EXAM

NOVEMBER 22-23, 2002

TAKE HOME SECTION

Name: Seymour Butz

Section Enrolled: (Circle) MWF 10 MWF 11 TR 11 TR 12:30

Social Security Number: 234567891

Throughout this exam **show your work!** Please indicate clearly what sections of the problem you are answering! If you are following a rule like $E(ax) = aE(x)$, please state it! If you are using a formula, state it! If you answer a 'yes' or 'no' question, explain why!

Part I. Do all the Following (10 Points) **Show your work!**

My Social Security Number is 265398248. If I write it as below, so that, for example, $x_3 = 5$, Minitab tells me that the sample mean is 5.222 and the sample standard deviation is 2.682. (Please don't do these again!) Write your Social Security next to it in the same way and call it y.

	x
1	2
2	6
3	5
4	3
5	9
6	8
7	2
8	4
9	8

Compute the following, showing the steps clearly as if you had done it by hand. Do not tell me "That is what my calculator said," though you are welcome to use your calculator, Excel or Minitab to check your work:

1. The sample variance of y. (2).
2. The sample covariance between x and y. (2)
3. The sample correlation between x and y. (2)
4. Interpret the correlation (1)

Answers to questions 5) and 6) must be based on the mean, standard deviation, covariance and correlation that you found in questions 1-4. Do not recompute the answers after changing x or y !

5. If all the numbers in y rise by 20, (so that if y was [2, 3, 4, 5, 6, 7, 8, 9, 1], it is now [22, 23, 24, 25, 26, 27, 28, 29, 21]) what will the new mean and standard deviation of y be? What will be the correlation between x and y be now? (1.5)
6. If, instead, all the numbers in y are multiplied by 3.5, (so that if y was [2, 3, 4, 5, 6, 7, 8, 9, 1], it is now [7, 10.5, 14, 17.5, 21, 24.5, 28, 31.5, 3.5]) what will the new mean and standard deviation of y be? What will be the correlation between x and y be now? (1.5)

Worksheet

obs	x	x^2	y	y^2	xy
1	2	4	2	4	4
2	6	36	3	9	18
3	5	25	4	16	20
4	3	9	5	25	15
5	9	81	6	36	54
6	8	64	7	49	56
7	2	4	8	64	16
8	4	16	9	81	36
9	8	64	1	1	8
	47	303	45	265	227

$$\bar{x} = \frac{\sum x}{n} = \frac{47}{9} = 5.22222$$

$$s_x^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{303 - 9(5.22222)^2}{8} = 7.19444 \quad s_x = 2.68225$$

$$\bar{y} = \frac{\sum y}{n} = \frac{45}{9} = 5.00000$$

$$s_y^2 = \frac{\sum y^2 - n\bar{y}^2}{n-1} = \frac{285 - 9(5.00000)^2}{8} = 7.50000 \quad s_y = 2.73861$$

$n = 9$, $\sum x = 47$, $\sum x^2 = 303$, $\sum y = 45$, $\sum y^2 = 285$ and $\sum xy = 227$

Note that the x and x^2 columns and their sums were not needed!

$$s_{xy} = \frac{\sum xy - n\bar{x}\bar{y}}{n-1} = \frac{227 - 9(5.22222)(5.00000)}{8} = \frac{-8}{8} = -1.00000$$ so

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{-1.00000}{\sqrt{7.19444}\sqrt{7.50000}} = -0.136135$$

Solution: From above:

1. The sample variance of y. $s_y^2 = 7.50000$
2. The sample covariance between x and y. $s_{xy} = -1.00000$
3. The sample correlation between x and y. $r_{xy} = -0.136135$
4. Interpretation. The negative sign of the covariance and correlation indicates that x and y tend to move in opposite directions. The correlation squared is about .019. On a zero to one scale, this is extremely weak.
5. If all the numbers in y rise by 20, what will the new mean and standard deviation of y be? What will be the correlation between x and y be now?

$E(cy + d) = cE(y) + d$ means that $E(1y + 20) = 1E(y) + 20$ works for sample means too, so that the sample mean rises by 20 to 25.0000.

And $Var(cy + d) = c^2 Var(y)$, so

$Var(y + 20) = (1)^2 Var(y) = (1)^2(7.50000) = 7.50000$. The standard deviation remains 2.73861.

Also, $Cov(ax + b, cy + d) = acCov(x, y)$, so

$$Cov(x + 0, y + 20) = 1(1)Cov(x, y) = 1(1)(-1.00000) = -1.00000$$

and if $w = ax + b$ and $v = cy + d$, $\rho_{wv} = (sign(ac))\rho_{xy}$. In this case

$w = 1x + 0$ and $v = 1y + 20$ $(sign(ac)) = (sign(1(1))) = +$, and we

started with a

correlation of $r_{xy} = -0.136135$, so the correlation between the two is -0.136135. In short, adding 20 to y has no effect.

6. If, instead, all the numbers in y are multiplied by 3.5, what will the new mean and standard deviation of y be? What will be the correlation between x and y be now? (1.5)

$E(cy+d) = cE(y)+d$ means that $E(3.5y+0) = 3.5E(y)+0$ works for sample means too, so that the sample mean, which was 5.00000 is multiplied by 3.5 to become 17.5.

And $Var(cy+d) = c^2 Var(y)$, so if our original standard deviation was $s_y^2 = 7.50000$ we find $Var(3.5y+0) = (3.5)^2 Var(y) = (3.5)^2(7.50000) = 19.75$.

Also, $Cov(ax+b, cy+d) = acCov(x,y)$, and $s_{xy} = -1.00000$ so $Cov(x+0, 3.5y+0) = 1(3.5)Cov(x,y) = 1(3.5)(-1.00000) = -3.5000$

and if $w = ax+b$ and $v = cy+d$, $\rho_{wv} = (sign(ac))\rho_{xy}$.

$(sign(ac)) = (sign(1(3.5))) = +$, so the correlation between the two is unchanged at

$r_{xy} = -0.136135$.

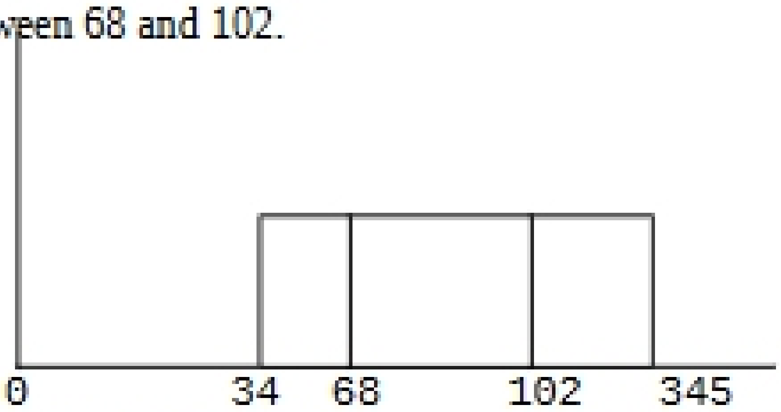
Part II: Take your Social Security number again, let g_1 be the 2nd and 3rd digits of the number, g_2 be the 2nd, 3rd and 4th digits and $g_3 = 2g_1$ and $g_4 = 3g_1$. (For example, my Social Security number is 265398248, so $g_1 = 65$, $g_2 = 653$, $g_3 = 2(65) = 130$, $g_4 = 3g_1 = 3(65) = 195$.

A jorcillator's lifespan (failure time) can be represented by a continuous uniform distribution between g_1 and g_2 years (My jorcillator has a lifespan between 65 and 653 years).

1. What is the probability that it lasts between g_3 and g_4 years? (1)
2. What is the probability that it lasts between g_3 and 1000 years? (1)
3. What is the mean life of such a jorcillator ? (1)
4. What is the standard deviation of the life of such a jorcillator ? (1)
5. If I have five such jorcillators, what is the probability that at least one lasts between g_3 and 1000 years? (1)

Solution: Seymour's Social Security number was 234567891, so that $g_1 = 34$, $g_2 = 345$, $g_3 = 2(34) = 68$, and $g_4 = 3g_1 = 3(34) = 102$. Our jorcillator has a lifespan from $c = 34$ to $d = 345$ years and we want $P(68 \le x \le 102)$. $\frac{1}{d-c} = \frac{1}{345-34} = \frac{1}{311} = .003215$.

1. So $P(68 \le x \le 102) = \frac{102-68}{345-34} = 34\frac{1}{311} = .1093$. In the diagram below (not to scale), shade the area between 68 and 102.



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The height of the box is $\frac{1}{d-c} = \frac{1}{345-34} = \frac{1}{311} = .003215$ and the base of the shaded area is