12/26/99 251y9941 **ECO251 QBA1** Name ________key_____

FINAL EXAM

DECEMBER 14, 1999

Part I. Do all the Following (20 Points) **Make Diagrams!**

A. $z \sim N(0,1)$

1. $P(z \leq 1.43) = P(z \leq 0) + P(0 \leq z \leq 1.43) = .5 + .4236 = .9236$ **Diagram! These are expected and will be provided in class for version 2.**

2. $P(1.43 \leq z \leq 3.24) = P(0 \leq z \leq 3.24) - P(0 \leq z \leq 1.43) = .4994 - .4236 = .0758$

3. $P(-1.43 \leq z \leq 1.43) = 2P(0 \leq z \leq 1.43) = 2(.4236) = .8472$

4. $P(-0.95 \leq z \leq -0.65) = P(-0.95 \leq z \leq 0) - P(-0.65 \leq z \leq 0) = .3289 - .2422 = .0867$

5. $z_{.97}$ This is the point with a probability of .97 above it or .03 below it. It is the 3rd percentile of z. So

$z_{.97} = -z_{.03}$. From the diagram $P(-z_{.03} \leq z \leq 0) = .4700$. the closest we can come to this probability using the Normal table is $P(0 \leq z \leq 1.88) = .4699$. So $z_{.97} = -z_{.03} \approx -1.88$.

Note that probabilities cannot be above 1 or below 0.

B. $x \sim N(1.5, 3)$ **.As usual, people made diagrams of X with zero in the middle. Make up your mind! If you are diagramming X, put the mean in the middle; if you are diagramming Z put zero in the middle.**

1. $P(x \le 1.43)$

$$= P\left(z \le \frac{1.43 - 1.5}{3}\right) = P(z \le -0.02) = P(z \le 0) - P(-0.02 \le z \le 0) = .5 - .0080 = .4920$$

Remember $z = \frac{x - \mu}{\sigma}$.

2. $P(1.43 \le x \le 3.24) = P\left(\frac{1.43 - 1.5}{3} \le z \le \frac{3.24 - 1.5}{3}\right) = P(-0.02 \le z \le 0.58)$

$$= P(-0.02 \le z \le 0) + P(0 \le z \le 0.58) = .0080 + .2190 = .2270$$

3. $P(-1.43 \le x \le 1.43) = P\left(\frac{-1.43 - 1.5}{3} \le z \le \frac{1.43 - 1.5}{3}\right) = P(-0.98 \le z \le -0.02)$

$$= P(-0.98 \le z < 0) - P(-0.02 \le z \le 0) = .3365 - .0080 = .3285$$

4. $P(-0.95 \le x \le -0.65) = P\left(\frac{-0.95 - 1.5}{3} \le z \le \frac{-0.65 - 1.5}{3}\right) = P(-0.82 \le z \le -0.72)$

$$= P(-0.82 \le z < 0) - P(-0.72 \le z \le 0) = .2939 - .2642 = .0297$$

5. $x_{.97}$ This is the point with a probability of .97 above it or .03 below it. It is the 3rd percentile of X. Because it is below the 50th percentile, which is the mean for the Normal distribution, the point we want will be below 1.5. On the previous page we found $z_{.97} = -z_{.03} \approx -1.88$. So

$$x_{.97} = \mu + z_{.97}\sigma$$
$$= 1.5 - 1.88(3) = -4.14.$$

To check to see if this is correct: $P(x \ge -4.14) = P\left(z \ge \frac{-4.14 - 1.5}{3}\right) = P(z \ge -1.88)$

$$= P(z \ge 0) + P(-1.88 \le z \le 0) = .5 + .4699 = .9699 \approx .97$$

II. (4 points-2 point penalty for not trying .) **Show your work!**

A manufacturer of printheads investigates the amount of time before a printhead fails. The data for a sample of 8(in millions of characters)are below.

x
1.5
1.4
1.2
1.6
1.3
1.3
1.4
1.5

Compute the sample standard deviation, s . Show your work. (4)

Solution:

Printhead lives

	x	x^2
1	1.5	2.25
2	1.4	1.96
3	1.2	1.44
4	1.6	2.56
5	1.3	1.69
6	1.3	1.69
7	1.4	1.96
8	1.5	2.25
	11.2	15.80

$$\bar{x} = \frac{\sum x}{n} = \frac{11.2}{8} = 1.40 \qquad s_x^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{15.80 - 8(1.40)^2}{7} = \frac{0.12}{7} = 0.017143$$

$$s_x = \sqrt{0.017143} = 0.13093 \qquad \left[s_{\bar{x}} = \frac{s_x}{\sqrt{n}} = \frac{0.13093}{\sqrt{8}} = 0.046291 \right]$$