

DECEMBER 15, 2000

Hint: Though you should never give 2 answers to a problem, never cross out the answer to a problem that you have given up on. I may see something worth partial credit in the answer.

Part I. Do all the Following (20 Points) **Make Diagrams!**

A. $z \sim N(0,1)$

1. $P(z \geq 2.93) = P(z \geq 0) - P(0 \leq z \leq 2.93) = .5 - .4983 = .0017$ **Diagram! These are expected and will be provided in class for version 2.**

Half of you didn't read this question: You assumed that it said $P(z \leq \dots)$, because that was on last year's exam.

2. $P(-3.42 \leq z \leq -0.19) = P(-3.42 \leq z \leq 0) - P(-0.19 \leq z \leq 0) = .4997 - .0753 = .4244$

3. $P(-1.07 \leq z \leq 1.07) = 2P(0 \leq z \leq 1.07) = 2(.3577) = .7154$

4. $P(1.07 \leq z \leq 2.57) = P(0 \leq z \leq 2.57) - P(0 \leq z \leq 1.07) = .4949 - .3577 = .1372$

5. The 93rd percentile of the distribution. $z_{.07}$ is the point with a probability of .07 above it or .93 below it. Since 93% is below this point and 50% is below zero, from the diagram $P(0 \leq z \leq z_{.11}) = .4300$.

The closest we can come to this probability using the Normal table is $P(0 \leq z \leq 1.48) = .4306$ So $z_{.07} \approx 1.48$.

Note that probabilities cannot be above 1 or below 0.

B. $x \sim N(1.2, 5)$.Remember $z = \frac{x - \mu}{\sigma}$. As usual, people made diagrams of X with zero in the middle. Make up your mind! If you are diagramming X , put the mean in the middle; if you are diagramming Z put zero in the middle.1. $P(x \geq 2.93)$

$$= P\left[z \geq \frac{2.93 - 1.2}{5}\right] = P(z \geq 0.35) = P(z \geq 0) - P(0 \leq z \leq 0.35) = .5 - .1368 = .3632$$

2. $P(-3.42 \leq x \leq -0.19) = P\left[\frac{-3.42 - 1.2}{5} \leq z \leq \frac{-0.19 - 1.2}{5}\right] = P(-0.92 \leq z \leq -0.28)$

$$= P(-0.92 \leq z \leq 0) - P(-0.28 \leq z \leq 0) = .3212 - .1103 = .2109$$

3. $P(-1.07 \leq x \leq 1.07) = P\left[\frac{-1.07 - 1.2}{5} \leq z \leq \frac{1.07 - 1.2}{5}\right] = P(-0.45 \leq z \leq -0.03)$

$$= P(-0.45 \leq z \leq 0) - P(-0.03 \leq z \leq 0) = .1736 - .0120 = .1616$$

4. $P(1.07 \leq x \leq 2.57) = P\left[\frac{1.07 - 1.2}{5} \leq z \leq \frac{2.57 - 1.2}{5}\right] = P(-0.03 \leq z \leq 0.27)$

$$= P(0 \leq z \leq 0.27) + P(0 \leq z \leq 0.03) = .1064 + .0120 = .1184$$

5. The 93rd percentile of the distribution.. $x_{.07}$ is the point with a probability of .07 above it or .93 below it. Because it is above the 50th percentile, which is the mean for the Normal distribution, the point we want will be above 1.2. On the previous page we found $z_{.07} \approx 1.48$. . So $x_{.97} = \mu + z_{.97}\sigma$
 $= 1.2 + 1.48(5) = 8.60$

To check to see if this is correct: $P(x \leq 8.60)$

$$= P\left[z \leq \frac{8.60 - 1.2}{5}\right] = P(z \leq 1.48) = P(z \leq 0) + P(0 \leq z \leq 1.48) = .5 + .4306 = .9306 \approx .93$$

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II. (4 points-2 point penalty for not trying .) **Show your work!**

We are investigating the cost of a business trip for people in the financial industry. The data for a sample of 6 are below.

X
1433
1225
1433
1573
1333
941

Compute the sample standard deviation, S . Show your work. (4)

Solution:

	X	x^2
1	1433	2053489
2	1225	1500625
3	1433	2053489
4	1573	2474329
5	1333	1776889
6	<u>941</u>	<u>885481</u>
	7938	10744302

$$\bar{x} = \frac{\sum x}{n} = \frac{7938}{6} = 1323$$

$$s_x^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{10744302 - 6(1323)^2}{5} = \frac{242328}{5} = 48465.6$$

$$s_x = \sqrt{48465.6} = 220.1490$$

How about those jokers who are still trying to compute $\sum (x - \bar{x})^2$ by computing $(\sum x - \bar{x})^2$! It didn't work last term and it won't work this term. I don't encourage using definitional formulas, but, if you insist on using them, I will be happy to give you a personal tutorial on how to do it. They are generally harder to use than computational formulas.