ECO 251 QBA1 Name **_KEY__________**
FINAL EXAM, Version 1 Class _______________
MAY 8, 2006

Part I. Do all the Following (14 Points) **Make Diagrams! Show your work! Illegible and poorly presented sections will be penalized.** Exam is normed on 75 points. There are actually 123+ possible points. If you haven't done it lately, take a fast look at ECO 251 - Things That You Should Never Do on a Statistics Exam (or Anywhere Else).

$x \sim N(13, 5.6)$

1. $P(0 \le x \le 28) = P\left(\frac{0-13}{5.6} \le z \le \frac{28-13}{5.6}\right) = P(-2.32 \le z \le 2.68)$

$= P(-2.32 \le z \le 0) + P(0 \le z \le 2.68) = .4898 + .4963 = .9861$

For $x \sim N(13, 5.6)$ *make a Normal curve centered at 13 and shade the area from 0 to 28; for* z *make a Normal curve centered at zero and shade the area from -2.32 to 2.68. Since the diagrams show an area on both sides of the mean, you add.*

2. $F(12.00)$ (Cumulative Probability) $P(x \le 12) = P\left(z \le \frac{12-13}{5.6}\right) = P(z \le -0.18)$

$= P(z \le 0) - P(-0.18 \le z \le 0) = .5 - .0714 = .4286$

For $x \sim N(13, 5.6)$ *make a Normal curve centered at 13 and shade the area below 12; for* z *make a Normal curve centered at zero and shade the area below -0.18. Since the diagrams show an area below the mean that does not touch the mean, you subtract.*

3. $P(x \ge 28) = P\left(z \ge \frac{28-13}{5.6}\right) = P(z \ge 2.68) = P(z \ge 0) - P(0 \le z \le 2.68)$

$= .5 - .4963 = .0037$

For $x \sim N(13, 5.6)$ *make a Normal curve centered at 13 and shade the area above 28; for* z *make a Normal curve centered at zero and shade the area above 2.68. Since the diagrams show an area above the mean that does not touch the mean, you subtract*

4. $P(28 \le x \le 32) = P\left(\frac{28-13}{5.6} \le z \le \frac{32-13}{5.6}\right) = P(2.68 \le z \le 3.39)$

$= P(0 \le z \le 3.39) - P(0 \le z \le 2.68)$

$.4997 - .4963 = .0034$

For $x \sim N(13, 5.6)$ *make a Normal curve centered at 13 and shade the area above 28; for* z *make a Normal curve centered at zero and shade the area between 3.39 and 2.68. Since the diagrams show an area above the mean that does not touch the mean, you subtract*

5. $P(-3 \le x \le 3) = P\left(\frac{-3-13}{5.6} \le z \le \frac{3-13}{5.6}\right) = P(-2.68 \le z \le -1.79)$

$= P(-2.68 \le z \le 0) - P(-1.79 \le z \le 0)$

$= .4963 - .4663 = .0300$

For $x \sim N(13, 5.6)$ *make a Normal curve centered at 13 and shade the area between -3 and 3, both of which are below 13; for* z *make a Normal curve centered at zero and shade the area between -2.68 and -1.79. Since the diagrams show an area below the mean that does not touch the mean, you subtract.*

$x \sim N(13, 5.6)$

6. $x_{.23}$ (Find $z_{.23}$ first) .
Solution: Make a diagram. *The diagram for* z *will show an area with a probability of 100 - 23% = 77%. below* $z_{.23}$ *. The area below* $z_{.23}$ *is split by a vertical line at zero into two areas. The lower one has a probability of 50% and the upper one a probability of 77% - 50% = 27%. The upper tail of the distribution above* $z_{.23}$ *has a probability of 23%, so that the entire area above 0 adds to 27% + 23% = 50%.* From the diagram, we want <u>one</u> point $z_{.23}$ so that $P(z \leq z_{23}) = .77$ or $P(0 \leq z \leq z_{29}) = .2700$. If we try to find this point on the Normal table, the closest we can come is $P(0 \leq z \leq 0.74) = .2704$, but $P(0 \leq z \leq 0.73) = .2673$ not as close, but is acceptable. So $z_{.23} \approx 0.74$

Since $x \sim N(13, 5.6)$ *, the diagram for* x *would show 77% probability split in two regions on either side of 13 with probabilities of 50% below 13 and 27% above 13 and below* $x_{.23}$ *, and with 23% above* $x_{.23}$ *.* $z_{.23} \approx 0.74$, so the value of x can then be written $x_{.23} = \mu + z_{.23}\sigma \approx 13 + 0.74(5.6) = 13 + 4.144 = 17.144.$

To check this:

$$P(x \geq 17.144) = P\left(z \geq \frac{17.144 - 13}{5.6}\right) = P(z \geq 0.74) = P(z \geq 0) - P(0 \leq z \leq 0.74)$$
$$= .5000 - .2704 = .2296 \approx .23.$$

7. A symmetrical region around the mean with a probability of 23%. [14, 14]
Solution: Make a diagram. *The diagram for* z *will show a central area with a probability of 23%. It is split in two by a vertical line at zero into two areas with probabilities of 11.5%. The tails of the distribution each have a probability of 50% - 11.5% = 38.5%.* From the diagram, we want two points $z_{.385}$ and $z_{.615}$ so that $P(z_{.615} \leq z \leq z_{.385}) = .2300$. The upper point, $z_{.385}$ will have $P(0 \leq z \leq z_{.385}) = \frac{23\%}{2} = .1150$, and by symmetry $z_{.615} = -z_{.385}$. From the interior of the Normal table the closest we can come to .1150 is $P(0 \leq z \leq 0.29) = .1141$, which is slightly too low. The next best point would be 0.30 since $P(0 \leq z \leq 0.30) = .1179$. We can say $z_{.385} \approx 0.29$, and our 23% symmetrical interval for z is -0.29 to 0.29.

Since $x \sim N(13, 5.6)$ *, the diagram for* x *(if we bother) will show 23% probability split in two 11.5% regions on either side of 13, with 38.5% above* $x_{.385}$ *and 38.5% below* $x_{.855}$ *.* The interval for x can then be written $x = \mu \pm z_{.385}\sigma \approx 13 \pm 0.29(5.6) = 13 \pm 1.624$ or 11.376 to 14.624.
To check this:

$$P(11.376 \leq x \leq 14.624) = P\left(\frac{11.376 - 13}{5.6} \leq z \leq \frac{14.624 - 13}{5.6}\right) = P(-0.29 \leq z \leq 0.29)$$

$$= 2P(0 \leq z \leq 0.29) = 2(.1141) = .2281 \approx 23\%$$

II. (10 points+, 2 point penalty for not trying part b.) **Show your work! Mark individual sections clearly.**

Wynn, Anthony and Avronovic give us the following data for a sample of eight professional golfers. Ea or (x) is earnings in thousands of dollars and SA or (y) is average score.

Row	Ea(x)	SA(y)
1	71.6	71.50
2	55.8	72.75
3	147.4	71.34
4	117.4	71.27
5	112.3	70.95
6	82.7	71.65
7	22.8	72.49
8	58.6	71.46

In order to speed things up, I have computed the sum of the first seven observations.

$\sum_{i=1}^{7} x = 610.000$, $\sum_{i=1}^{7} y = 501.950$, $\sum_{i=1}^{7} x^2 = 63720.1$, $\sum_{i=1}^{7} y^2 = 35996.0$ and $\sum_{i=1}^{7} xy = 43607.4$.

Calculate the following:

a. The sums that you will need to calculate whatever parts you do. (1 point if you don't quit at b) Make sure that I can tell how you did these sums.

b. The sample standard deviation s_y of average score. (2)

c. The sample covariance s_{xy} between x and y. (2)

d. The sample correlation r_{xy} between x and y. (2)

e. Given the size and sign of the correlation, what conclusion might you draw on the relation between x and y? (1) Can you guess why the correlation isn't stronger?

f. Assume that the earnings of the golfers were 15% lower ($w = .85x$). Find $\bar{w}$ (the sample mean of earnings), s_w^2, s_{wy} and r_{wy}. Use only the values you computed in a-d and rules for functions of x and y to get your results. If you state the results without explaining why, or change x_1 and x_2 and recompute the results, you will receive no credit. (4).

g. Do an 80% confidence interval for the population average score of professional golfers. (2)

[14, 28]

Solution: Here is the table that you should have generated.

Row	Ea(x)	SA(y)	x^2	y^2	xy
1	71.6	71.50	5126.6	5112.25	5119.4
2	55.8	72.75	3113.6	5292.56	4059.5
3	147.4	71.34	21726.8	5089.40	10515.5
4	117.4	71.27	13782.8	5079.41	8367.1
5	112.3	70.95	12611.3	5033.90	7967.7
6	82.7	71.65	6839.3	5133.72	5925.5
7	22.8	72.49	519.8	5254.80	1652.8
8	58.6	71.46	3434.0	5106.53	4187.8
	668.6	573.41	67154.1	41102.57	47794.9

Of course, you we only responsible for the last two lines.