

ECO251 QBA1  
 SECOND EXAM  
 March 28, 2007  
 TAKE HOME SECTION

Name: \_\_\_\_\_  
 Student Number: \_\_\_\_\_

Throughout this exam **show your work! Neatness counts!** Please indicate clearly what sections of the problem you are answering and what formulas you are using.

Part II. Do all the Following (12 Points) **Show your work!**

1. Seymour Butz's student number is 976500

Take the following set of numbers:

100  
 200  
 300  
 400  
 500  
 600  
 700

and use your Student Number to provide the last digit of the last six numbers, so that, for Seymour, the numbers would become 100, 209, 307, 406, 505, 600, 700. Compute a sample standard deviation for the resulting numbers. Do not invent any probabilities. (2- 2point penalty for not doing)

2. Take exactly the same numbers as before and assume that they have the following probabilities: .05, .07, .08, .10, .10, .12, .48. Show that this represents a valid distribution and compute the population standard deviation from the distribution. (2). Find  $P(x \leq 500)$  and the median of the distribution. (1)

Seymour would now have:

$x$	$P(x)$
100	.05
209	.07
307	.08
406	.10
505	.10
600	.12
700	.48

[5]

3. As everyone knows, a jorcillator has two components, a phillinx and a flubberall. It seems that the jorcillator only works as long as one component works (so that it fails in the first month only if both components fail).

The probability of the phillinx failing is given by a continuous uniform distribution between  $c_1$  and  $d_1$  with  $c_1 = 1$  and  $d_1 = 4 + 0.2g$ , where  $g$  is the last digit of your student number. For example, if the life of the phillinx is represented by  $x_1$ , then the chance of the phillinx failing in the 3<sup>rd</sup> month is  $P(2 \leq x_1 \leq 3)$  and the probability of it failing after the third month is  $P(x_1 \geq 3)$ . For example: Ima Badrisk has the number 375292, so the top of her distribution is  $d_1 = 4 + 0.2(2) = 4.4$ . Depending on your number  $d_1$  could go up to 5.

The probability of the flubberall failing is given by the continuous uniform distribution between  $c_2$  and  $d_2$  with  $c_2 = .9 - 0.1g$  and  $d_2 = 4$ . For example: Ima Badrisk has the number 375292, so the bottom of her distribution is  $c_2 = .9 - 0.05(2) = .8$ . If  $x_2$  represents the life of the flubberall, the probability of the flubberall failing in the first month is  $P(0 \leq x_2 \leq 1)$ , the probability of the flubberall failing in the second month is  $P(1 \leq x_2 \leq 2)$  etc. We will divide time into four periods with the last period being 'beyond the third month.' The jorcillator is guaranteed to fail in one of the four periods. Failure of components is assumed to be independent, so if the probability of the phillinx failing in the first month is .1, and the probability of the flubberall failing in the first month is .7, the probability of both components failing in the first month is  $(.1)(.7) = .07$  (This is not the necessarily the probability that the jorcillator will fail in the first month!)

In order to maintain my sanity, use the following events.

Failure of the phillinx in period 1, 2, 3, 4 are events  $A_1, A_2, A_3,$  and  $A_4$ .

Failure of the flubberall in period 1, 2, 3, 4 are events  $B_1, B_2, B_3,$  and  $B_4$ .

Failure of the jorcillator in period 1, 2, 3, 4 are events  $C_1, C_2, C_3,$  and  $C_4$ .

- What is the probability that the phillinx will fail in month 1? Month 2? Month 3? After Month 3? (1.5)
- What are the mean and standard deviation of the phillinx's failure time? (1.5)
- What is the probability that the jorcillator will fail in the first month? (2)
- What is the probability that the jorcillator will fail in the second month? (1)
- What is the probability that the jorcillator will fail in the third month? (1)
- What is the probability that the jorcillator will last beyond 3 months? (1)
- Find the probability that the jorcillator and the Phillinx both fail in the second month (1)
- Find the probability that the phillinx fails in the second month, given that the jorcillator fails in the second month i.e.  $P(A|C)$  (1)
- Demonstrate Bayes' rule by showing how to get the probability that the jorcillator fails in the second month, given that the phillinx fails in the second month, from your result in h). (1)