9/7/2014

8.4

HW Problem $V_1(T) = 4\sin(377T + 25°)$ V

$i_1(T) = 0.05\cos(377T - 20°)\ A_1$

$i_2(T) = -0.1\sin(377T + 45°)\ A_2$

$V_1(T) = 4\cos(377T - 65°)\ V_1$

$i_1(T) = 0.05\cos(377T - 20°)\ A_1$

$i_2(T) = 0.1\cos(377T + 135°) A$

$V_1(T)$ leads $i_1(T)$ by $\Phi_1 - \Phi_2$ so i_1 leads v_1 by 45°

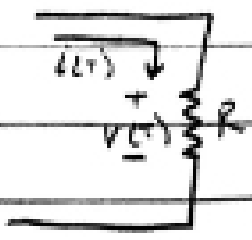
$= -65 - (-20) = -45°$

$V_1(T)$ leads $i_2(T)$ by $\Phi_1 - \Phi_2$ so i_2 lead v_1 by 200°

$= -65 - 135 = -200°$

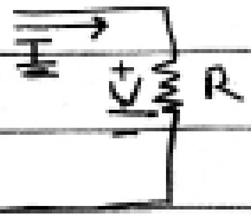
Resistor:

Time domain:



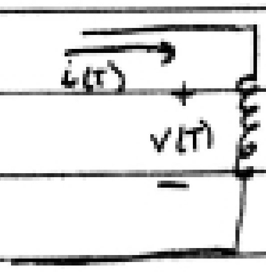
$\frac{V(T)}{i(T)} = R$

Phasor domain:



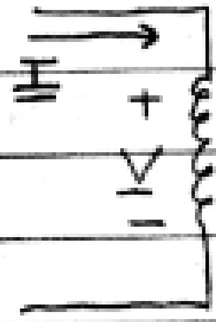
$\frac{V}{I} = R$

Inductor:



$V(T) = L\frac{di}{dT}$

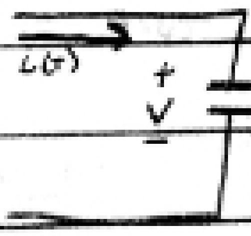
Impedance



JwL

$\frac{V}{I} = JwL$

Capacitor:



$i(T) = C\frac{dv}{dT}$

$-J\frac{1}{wC}$

$\frac{V}{I} = -J\frac{1}{wC}$

A. Impedance

1. The "resistance" of a network with R, L and C components
   - HAS units of ohms (Ω)
2. Define as a ratio

$$(\text{Impedance})\ \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m \angle \phi}{I_m \angle \beta}$$

3. $\mathbf{Z} = |Z| \angle \theta_z$

$$|Z| = \frac{V_m}{I_m} \qquad \phi_z = \frac{\angle \phi}{\angle \beta} = \angle \phi - \beta$$

4. No Time domain correspondance
   i.e. no $|Z| \cos(\omega t + \phi_z)$
5. Rectangular form of $\mathbf{Z}$

$$\mathbf{Z} = R + jX$$

Resistance (Real)   Reactance (Imaginary)

6. Converting to Polar form

$$|Z| = \sqrt{R^2 + X^2}$$

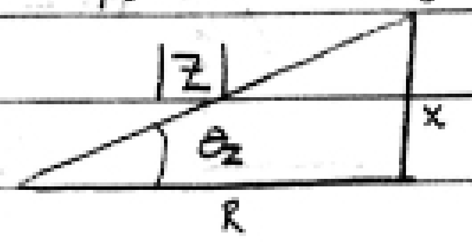
$$\phi_z = \tan^{-1} \frac{X}{R}$$

7. Converting to Rectangular

$$R = |Z| \cos \theta_z$$

$$X = |Z| \sin \theta_z$$

Impedance Triangle



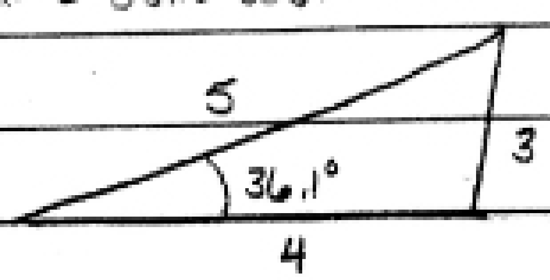
Example: The Terminal Voltage and current of a circuit are:

$$\underline{V} = 10 \angle 56.1^\circ \text{ V}$$

$$\underline{I} = 2 \angle 20^\circ \text{ A} \qquad \therefore Z = \frac{\underline{V}}{\underline{I}} = \frac{10 \angle 56.1}{2 \angle 20} = 5 \angle 36.1^\circ \ \Omega$$

$$R = 5 \cos(36.1) = 4$$

$$X = 5 \sin(36.1^\circ) = 3 \qquad \therefore \mathbf{Z} = 4 + 3j \ \Omega$$



9. Reactance, X

$\frac{V_L}{I_L} = j\omega L = jX_L$ $\quad X_L = \omega L$ → Inductive reactance

$\frac{V_C}{I_C} = -j\frac{1}{\omega C} = -jX_C$ $\quad X_C = \frac{-1}{\omega C}$ → Capacitive reactance

$Z = R + j(X_L + X_C)$

$= R + j\left(\omega L - \frac{1}{\omega C}\right)$

$\frac{1}{R} = G$ Conductance

$\frac{1}{Z} = Y$ admittance

$Y = |Y| \angle \theta_Y$

$Z = R + jX$

$Y = G + jB$

conductance, susceptance (reciprical of Reactance)

$Z = R + jx$

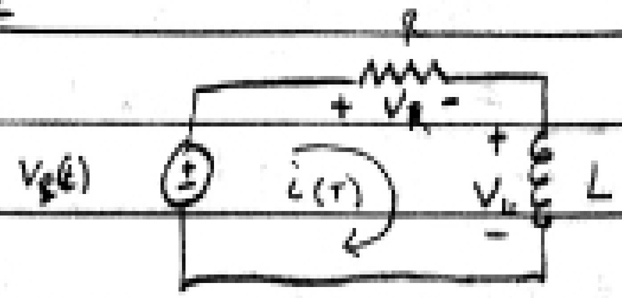
$\frac{1}{Z} = \frac{1}{R + jx} \cdot \frac{R - jx}{R - jx} = \frac{R - jx}{R^2 + x^2} = \frac{R}{R^2 + x^2} - j\frac{x}{R^2 + x^2}$

$Y = G + jB$

$G = \frac{R}{R^2 + x^2}$

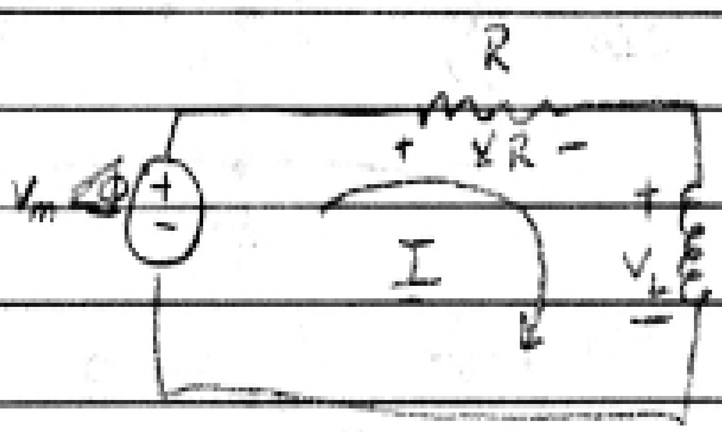
$B = \frac{-x}{R^2 + x^2}$

Example



$V_s(T) = V_m \cos(\omega T + \phi)$

$V_s(T) = V_R(T) + V_L(T)$



Find I

$V_m\angle\phi = RI + j\omega L I$

$I[R + j\omega L] = V_m\angle\phi$

$I = \frac{V_m\angle\phi}{R + j\omega L}$

$= \frac{V_m\angle\phi}{\sqrt{R^2 + \omega^2 L^2}\angle\theta}$

$I = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \angle \phi - \theta$

$\boxed{i(T) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega T + \phi - \theta)}$

$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$