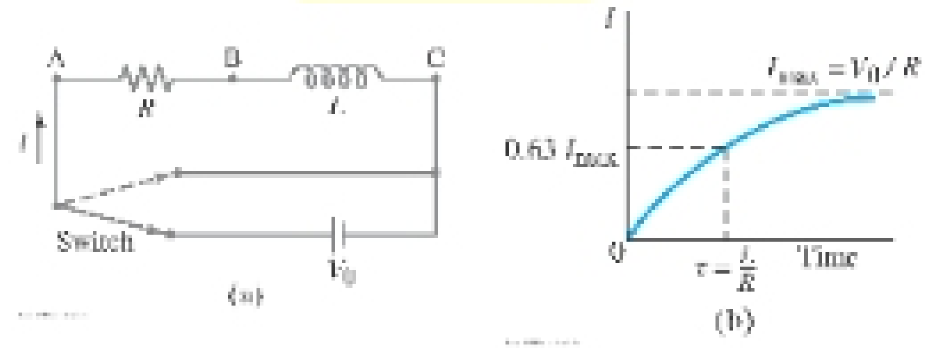
Physics 202, Lecture 18

Today's Topics

- Reminder
  - RL Circuits
  - RC Circuits
  - LC (RLC) Circuits and Electromagnetic Oscillations
- AC Circuits with AC Source
  - Phasors

Turn on RL Circuit (reminder)

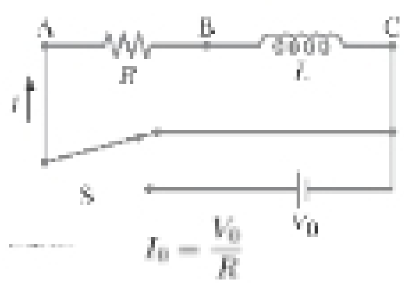
$$I = \frac{V_0}{R}(1 - e^{-\frac{t}{L/R}})$$

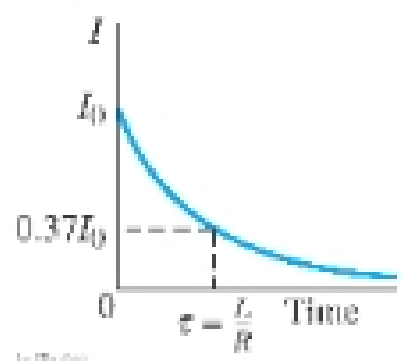


Note: the time constant is τ=L/R

Turn off RL Circuit (reminder)

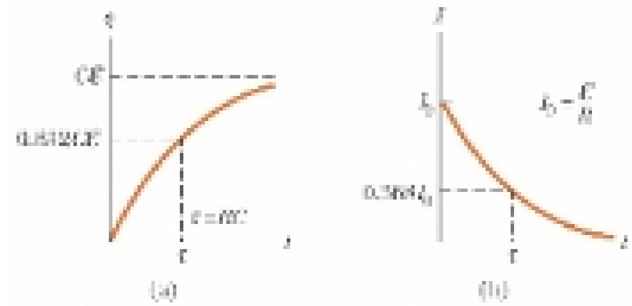
$$I = I_0 e^{-\frac{t}{L/R}}$$





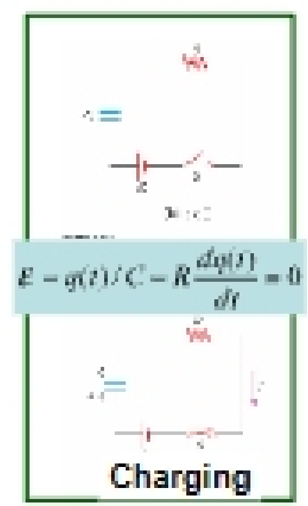
Note: the time constant is τ=L/R

Charging a Capacitor in RC Circuit



$$q(t) = \mathcal{E}C(1 - e^{-t/RC})$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$$



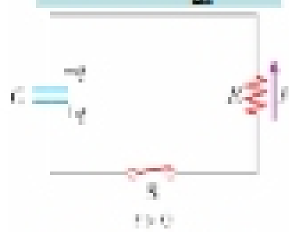
Note: τ≡ RC is called the time constant

Discharging a Capacitor in RC Circuit

$$q(t) = Qe^{-t/RC}$$

$$I(t) = -\frac{Q}{RC}e^{-t/RC}$$

$$q(t)/C + R\frac{dq(t)}{dt} = 0$$



discharging

Note the time constant τ=RC

LC Circuit and Oscillation

- Exercise: Find the oscillation frequency of a LC circuit

→ $$-q(t)/C - L\frac{dI(t)}{dt} = 0$$

→ $$q(t)/C + L\frac{d^2q(t)}{dt^2} = 0$$

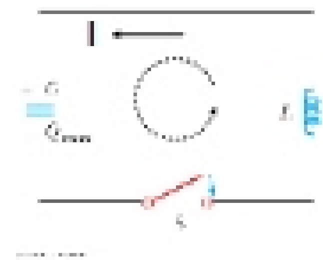
→ $$q(t) + \frac{1}{\omega_0^2}\frac{d^2q(t)}{dt^2} = 0$$

eq. of Harmonic Oscillation

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$q = Q_{max}\cos(\omega_0 t + \varphi)$$

$$I = -\omega_0 Q_{max}\sin(\omega_0 t + \varphi)$$



Total Energy is conserved

Show that energy is conserved in LR circuit:

$$E = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$
$$= \frac{1}{2C}Q_0^2\cos^2(\omega_0 t + \varphi) + \frac{1}{2}L\left[-\omega_0 Q_0\sin(\omega_0 t + \varphi)\right]^2$$
$$= \frac{Q_0^2}{2C}\cos^2(\omega_0 t + \varphi) + \frac{1}{2}\omega_0^2 LQ_0^2\sin^2(\omega_0 t + \varphi)$$

Now use $\omega_0 = 1/\sqrt{LC}$

$$E = \frac{Q_0^2}{2C}\cos^2(\omega_0 t + \varphi) + \frac{1}{2LC}LQ_0^2\sin^2(\omega_0 t + \varphi)$$
$$= \frac{Q_0^2}{2C}\left[\cos^2(\omega_0 t + \varphi) + \sin^2(\omega_0 t + \varphi)\right]$$
$$= \frac{Q_0^2}{2C}$$

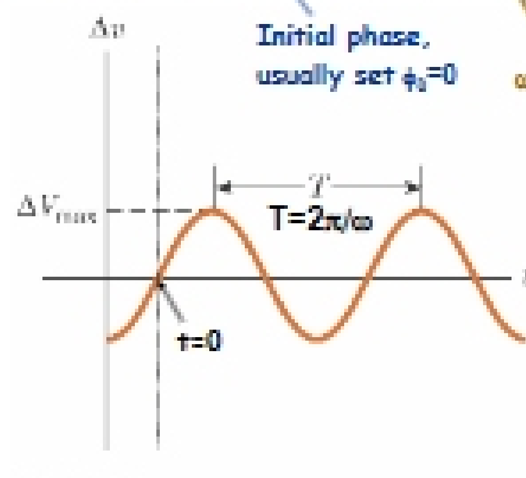
independent of time

AC Power Source

- $\Delta V = \Delta V_{max}\sin(\omega t + \phi_0) = \Delta V_{max}\sin(\omega t)$

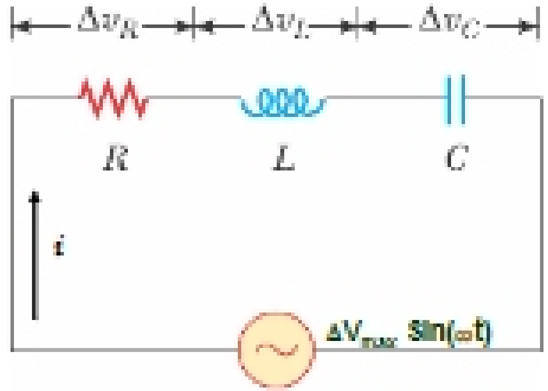
Initial phase, usually set ϕ_0=0

ω: angular frequency ω=2πf



AC Circuit

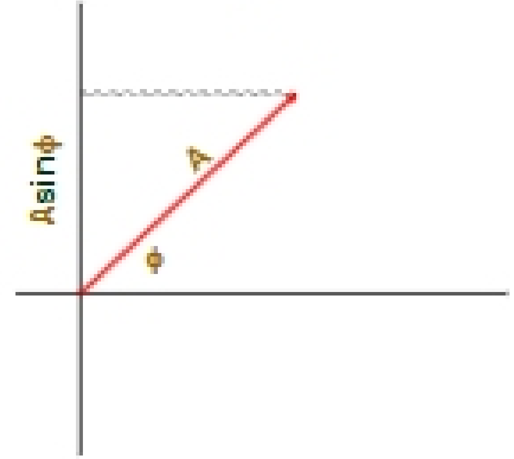
- Find out current i and voltage difference ΔV_R, ΔV_L, ΔV_C.



Notes:
- Kirchhoff's rules still apply !
- A technique called phasor analysis is convenient.

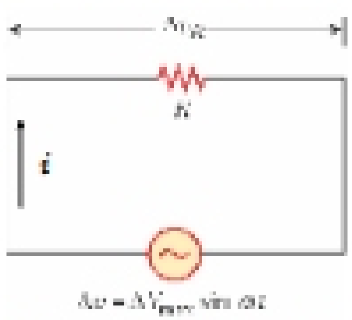
Phasor

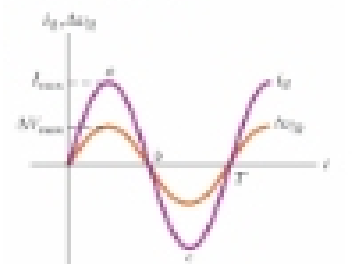
- A sinusoidal function $y = A\sin\phi$ can be represented graphically as a phasor vector with length A and angle ϕ (w.r.t. to horizontal)



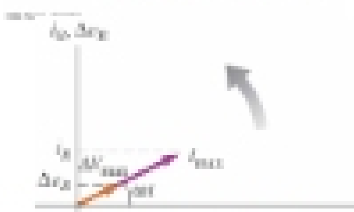
Resistors in an AC Circuit

- Ohm's Law: $\Delta V = IR$ at any time





Function view



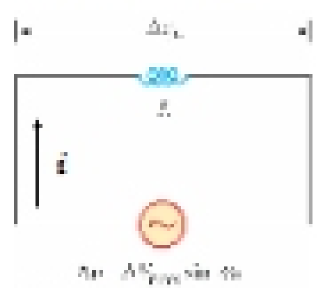
Phasor view

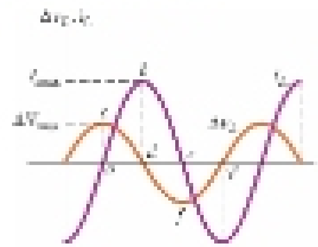
$i_R = \Delta V/R = I_{max}\sin\omega t$, $I_{max} = \Delta V_{max}/R$

➔ The current through an resistor is in phase with the voltage across it

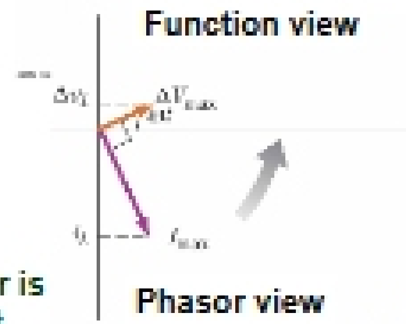
Inductors in an AC Circuit

- $\Delta V - L\,di/dt = 0$





Function view



Phasor view

➔ $i_L = I_{max}\sin(\omega t - \pi/2)$
$I_{max} = \Delta V_{max}/X_L$,
$X_L = \omega L$ → inductive reactance
➔ The current through an inductor is 90° behind the voltage across it.