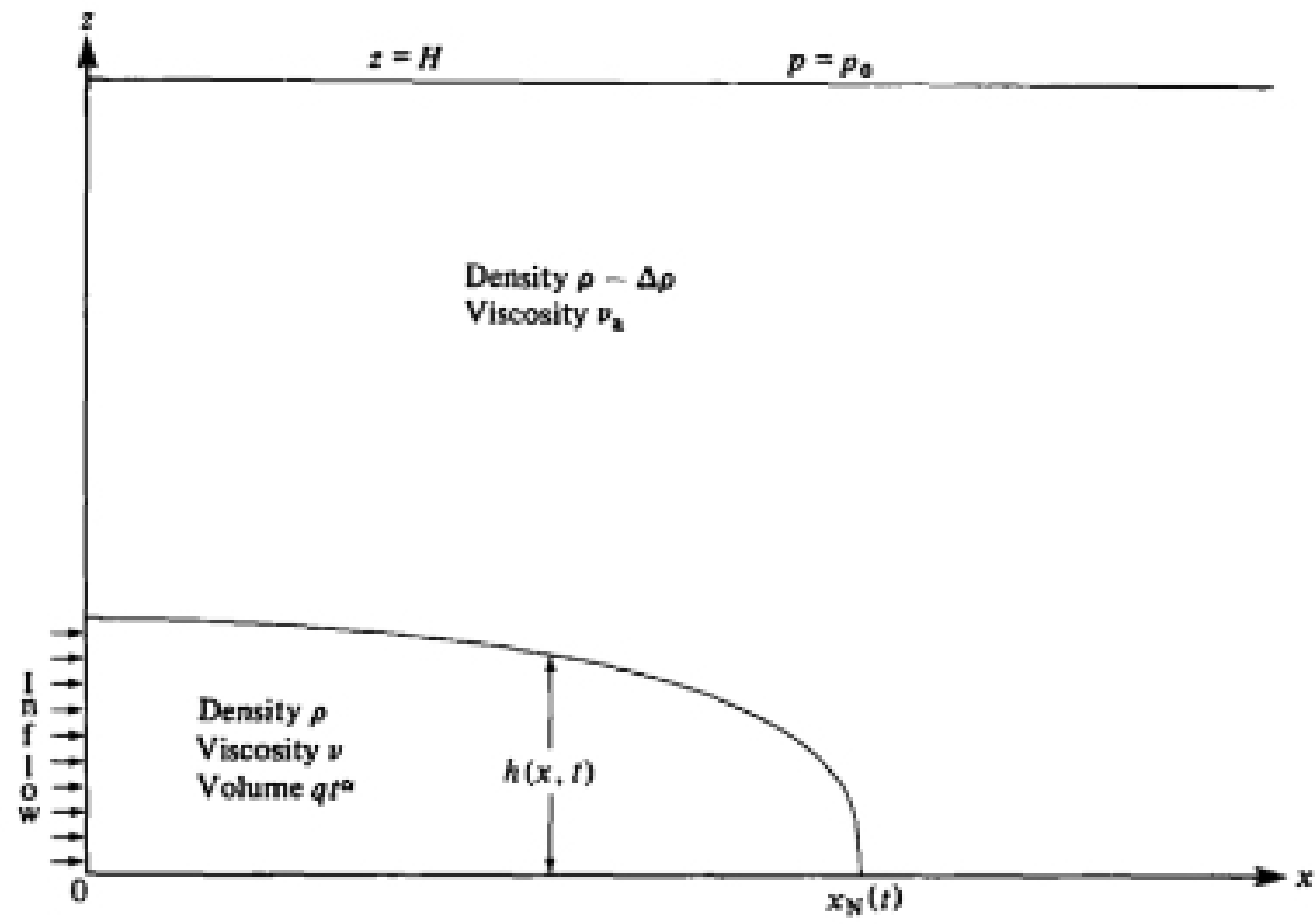
MENG 463 / ENAS 704
Fall 2010

Problem Set 8

Gravity currents are flows driven by horizontal density gradients. They are extremely common in geophysical settings: examples include thunderstorm outflows, mudslides, avalanches, and pyroclastic flow. Here we'll examine some of the flow properties of *viscous* gravity currents; picture the spreading of a blob of molasses. We will work in two dimensions. For this problem, refer to the following figure:



From H. E. Huppert, *J. Fluid Mech.* **121**, 43–58 (1982).

1. Why is this type of flow is called a gravity current? What are the driving forces? How do you expect the flow to evolve? What is the meaning of the exponent α?

2. We can approximate a viscous gravity current as a Stokes flow, so that the equation of motion is a balance between pressure gradients and viscous forces. Assuming that the pressure is hydrostatic in the vertical direction, what is the pressure distribution?

3. Show that the horizontal velocity of the current is given by

$$u(x, z, t) = -\frac{1}{2}\frac{g'}{\nu}\frac{\partial h}{\partial x} z(2h - z),$$

where $g' = g\Delta\rho/\rho$ is the reduced gravity. Clearly state the assumptions you make about the physical system.

4. Aside from the inflow at the far left of the figure, the volume of fluid in the body of the current cannot change. Use this fact to show that the equation of motion for the height h of the current is

$$\frac{\partial h}{\partial t} - \frac{1}{3}\frac{g'}{\nu}\frac{\partial}{\partial x}\left(h^3\frac{\partial h}{\partial x}\right) = 0.$$

What are the boundary conditions?

5. This equation admits a self-similar solution. We'll approach this solution in a slightly different way from the last problem set. Similarity solutions cannot depend on the independent variables non-algebraically. So, we can write similarity forms

$$\hat{\eta} = Axt^{-n}$$

and

$$h(x,t) = Bt^{-m}\hat{F}(\hat{\eta}/\hat{\eta}_N),$$

where A, B, m and n are to be determined and $\hat{\eta}_N$ is the value of $\hat{\eta}$ at the nose of the current. Show that the equation for h can be written as

$$(\hat{F}^3\hat{F}')' + \frac{1}{5}(3\alpha+1)y\hat{F}' - \frac{1}{5}(2\alpha-1)\hat{F} = 0,$$

where $y = \hat{\eta}/\hat{\eta}_N$ and primes denote differentiation with respect to y. What are A, B, m, and n?