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Electromechanical Dynamics

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Chapter 9

SIMPLE ELASTIC CONTINUA

9.0 INTRODUCTION

The study of the effects of motion on electric and magnetic fields (Chapter 7) and of electromagnetic force densities (Chapter 8) provides the background necessary for an introduction to the electromechanics of continuous media. To someone familiar with the dynamics of continuous media this is a pretentious statement, for it implies that the description of distributed mechanical systems requires only a minor addition to the largely electromagnetic considerations so far introduced. In general, this is far from the case; for example, does the mechanical medium consist of a solid or a fluid? In either case the equations of motion vary considerably with the particular fluid or solid under study. These equations generally involve three-dimensional deformations, hence are likely to be at least as complicated as the electromagnetic field equations if not more so.

Fortunately, many of the most significant and practical interactions with continuous media can be modeled in terms of one or two-dimensional structures that not only retain the salient features of the three-dimensional dynamics but represent idealizations that we should like to approach in practice. In this and the next chapter attention is confined to situations in which the mechanical side of the electromechanical problem takes the form of one of two simple models: the thin rod subject to longitudinal motions and wires and membranes undergoing transverse motions. The derivation of the one- and two-dimensional equations of motion for these simple cases serves to illustrate the essential steps required to write the more general expressions for elastic media and fluids, as undertaken in Chapters 11 and 12. At the same time the continuum electromechanical dynamics studied in this and the next chapter give a preview of types of dynamics found in acoustics, fluid dynamics, electron beam-plasma dynamics, magnetohydrodynamics, electrohydrodynamics, and microwave magnetics.

In this chapter the discussion is limited to electromechanical interactions with continuous media that occur through boundary conditions representable by terminal pairs. In Chapter 10 we consider physical situations in which the electromechanical coupling is itself distributed and in which our lumped parameter concept of a terminal pair can no longer account for the coupling.

9.1 LONGITUDINAL MOTION OF A THIN ROD

Longitudinal motion of a thin elastic rod provides a logical first topic in discussing the dynamics of elastic continua. This is true because we emphasize the wavelike nature of the dynamics; and in a thin rod longitudinal waves have a particularly simple form. As we shall see, waves in a thin rod can propagate without changing their shapes; hence they can be understood by means of comparatively simple mathematical techniques. This distortion-free behavior of the thin rod is used in applications such as acoustic delay lines and electromechanical filters in which the properties of the electromechanical system are especially attractive. We discuss some applications later in this section.

To describe longitudinal motion in an elastic rod we must make a mathematical model. This process consists essentially of two steps: (a) a mathematical description of force equilibrium for a small element of the rod and (b) a description of the elastic property of the rod.

We consider the long thin rod shown in Fig. 9.1.1*a*. The rod has a uniform cross section of area A perpendicular to the longitudinal (x_1)-direction. We apply forces in the x_1-direction and observe motion in the x_1-direction. By "thin" we mean that the dimensions of the rod perpendicular to x_1 are small enough that effects of any transverse motion are negligible. The

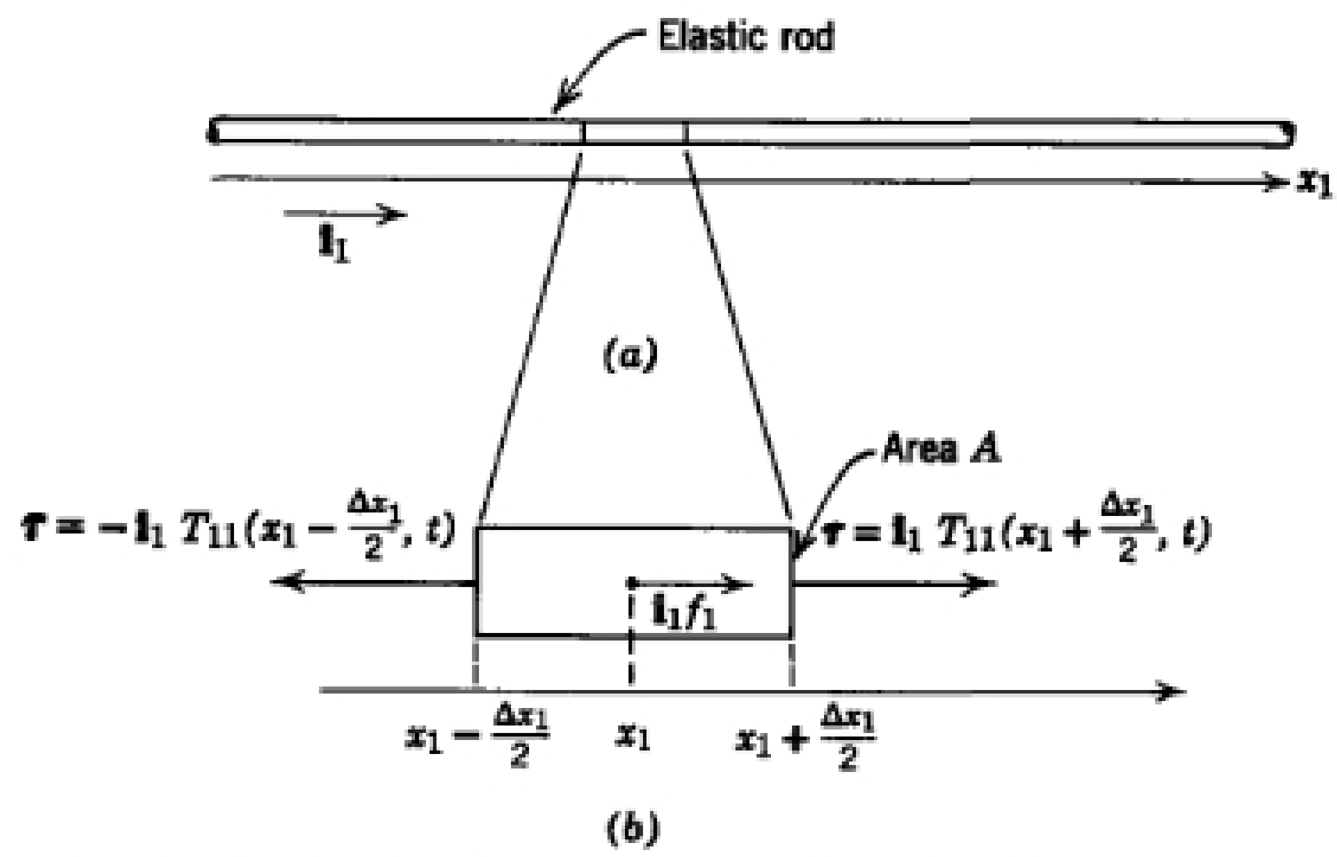

Fig. 9.1.1 Thin elastic rod with axis in the x_1-direction and uniform cross section of area A: (*a*) the rod; (*b*) force and tractions applied to an element of length Δx_1 centered at x_1.