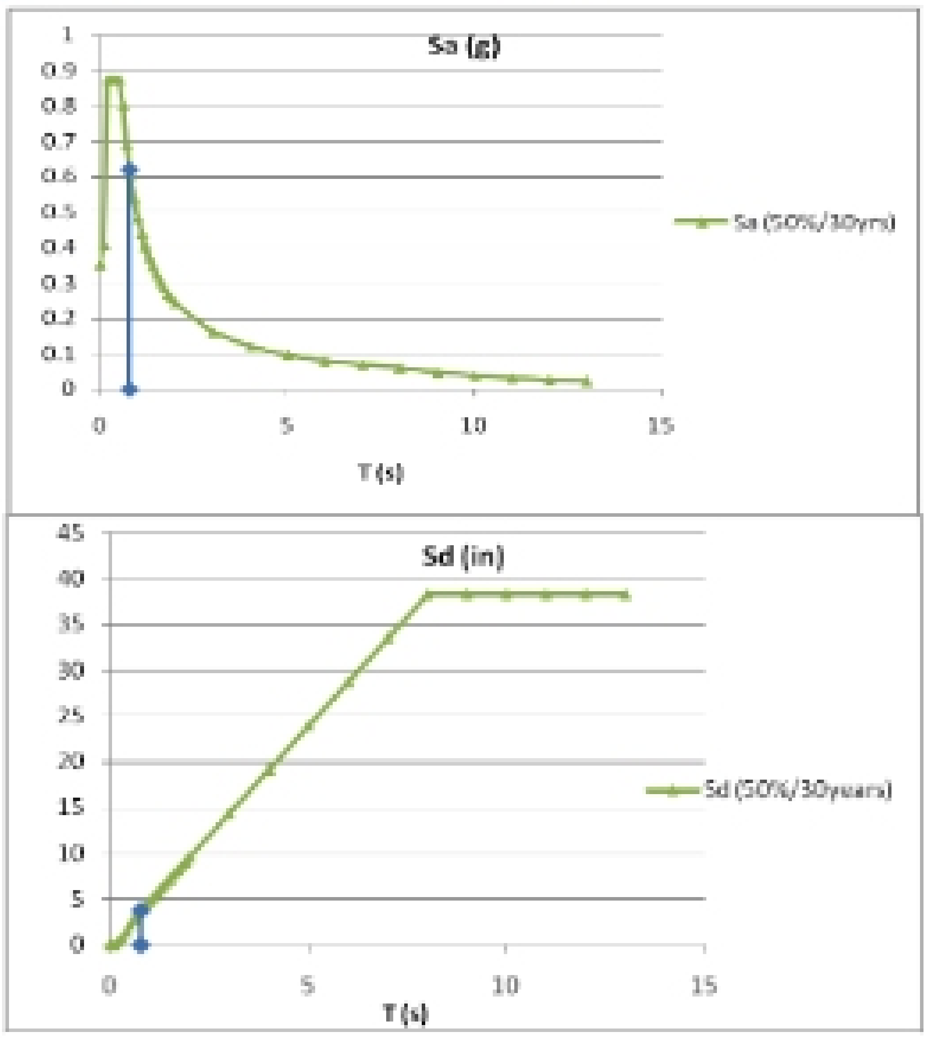
Problem 13 – Estimation of Elastic Design Forces and Displacements

In this problem we examine the demands on our building structure subjected to the frequent event, corresponding to the 50% in 30 yr discussed in problem 11. Since one of the performance goals established is for the structure to remain in the elastic range of response under this event, we can directly use the elastic spectrum obtained in Problem 11 to compute the force and displacement demand. For a first-mode period of 0.79 sec, we obtain the following spectral values: $A_1=S_a(T_1,\xi=3\%)=0.62g$ and $D_1=S_d(T_1,\xi=3\%)=3.78$ in.

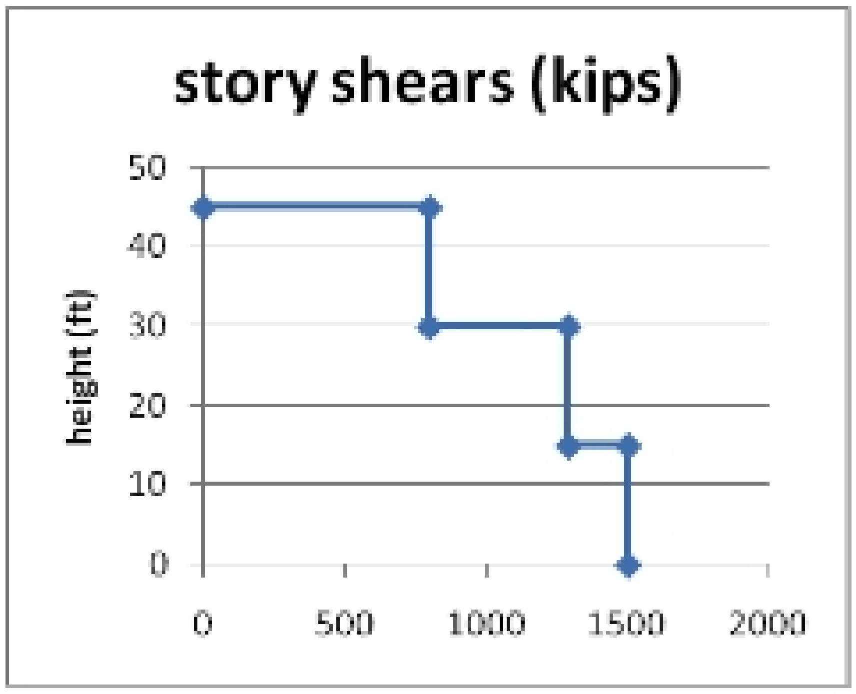


a. Computation of demand forces and displacements for the frequent event:
   i. The story shears are obtained as follows (obtained for one frame, not the entire building):
   Story forces: $\underline{f_1} = \underline{S_1} A_1$, $\underline{S_1} = \Gamma_1 \underline{m} \underline{\phi_1}$, where:

$$\underline{m} = \begin{bmatrix} m_{floor}/2 & 0 & 0 \\ 0 & m_{floor}/2 & 0 \\ 0 & 0 & m_{roof}/2 \end{bmatrix} = \begin{bmatrix} 2.525 & 0 & 0 \\ 0 & 2.525 & 0 \\ 0 & 0 & 2.590 \end{bmatrix}, \ \underline{\phi_1} = \begin{bmatrix} .27 \\ .63 \\ 1 \end{bmatrix}, \ \underline{\Gamma_1} = 1.29$$

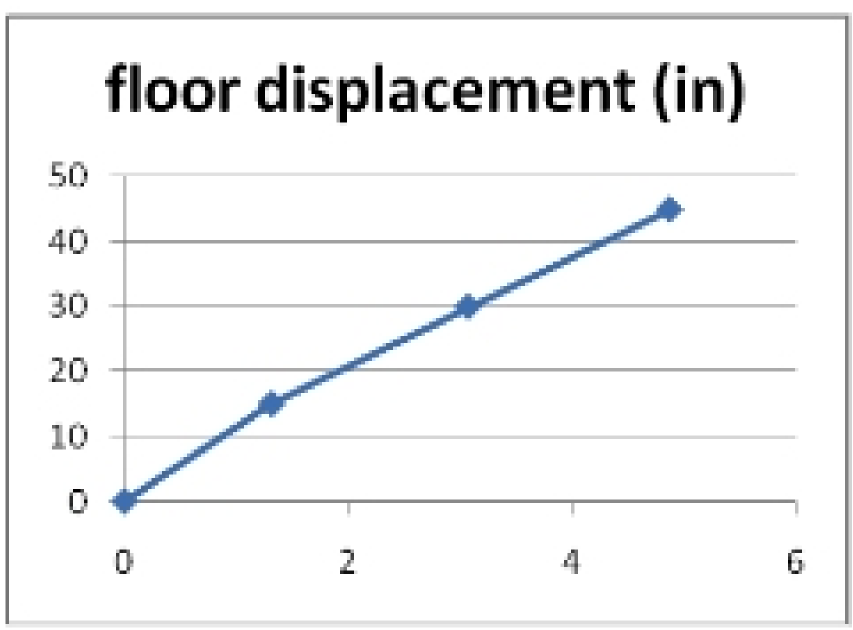
   This gives story shears:

Story	Story Shear (kips)
1	1500
2	1290
3	799



ii. The expected floor displacements and inter-story drifts are obtained as follows (for one frame or the entire building):
$v_{j1} = u_{j1} - u_{(j-1)1}$; $\underline{u}_1 = \underline{\phi}_1 \Gamma_1 D_1$, $\Delta_{j1} = v_{j1}/h_j$
This gives:

Story	Floor displacement (in)	Inter-story drift (in)	Interstory drift index
1	1.32	1.32	0.0073
2	3.07	1.76	0.0098
3	4.88	1.80	0.0100



b. Discussion of results:
   i. A typical upper-bound value of 0.25% rad is established for the protection of partitions. With a maximum drift demand of 1.0% estimated for the building for the frequent event (which is 4 times that limit), we can clearly expect damage to displacement-sensitive nonstructural components in the building under the frequent earthquake event.

ii. Recognizing that the first-mode period of the structure lies in the constant velocity range, we can observe that displacements are linearly related to the period. Thus, a reduction of 10% in the displacement demand and inter-story drifts corresponds to a reduction of 10% in the spectral displacement and fundamental period. Since we have $S_d \propto T \propto \sqrt{M/K}$, a reduction of 21% in the mass or an increase of 21% in stiffness will produce the desired effect.

From table 1-6 of FEMA 356, we have a damping coefficient for the constant velocity range B_1 of 0.8 and 1.0 for 2% and 5% effective viscous damping in the structural system, respectively. The spectral values have the following relation (when mass and stiffness are fixed values): $S_d \propto S_a \propto B_1^{-1}$, a reduction of 10% in the spectral values implies an increase of 11% in the damping coefficient B_1 and effective viscous damping β. ($B_1 = 0.8 \times 1.11 = 0.89$, which corresponds to a viscous damping $\beta = 3 \times 1.11 \approx 3.35\%$).

iii. A reduction in acceleration is directly proportional to the spectral acceleration. Since in the constant velocity range $S_a = S_{v,max}/T$, where $S_{v,max}$ is a constant, a reduction of 10% in the spectral acceleration values would require an increase in the mass and a reduction in the stiffness. Note that an increase in mass will result in an increase in the story forces and shears, despite a decrease in acceleration values ($V_b = MS_a = M \cdot 2\pi S_{v,max}/T = S_{v,max}\sqrt{MK}$).