Chapter 8

FIELD DESCRIPTION OF MAGNETIC AND ELECTRIC FORCES

8.0 INTRODUCTION

Chapter 7 is restricted to the effects of mechanical motion on magnetic and electric fields. In general, electromechanical interactions involve effects on the mechanical system from the electromagnetic fields as well. These arise from the mechanical forces of electrical origin.

In Chapters 3 through 6 we were concerned with total forces acting on rigid bodies. In systems in which the mechanical medium must be represented by a deformable continuum the details of the force distribution must be known. Hence in continuum electromechanics we are concerned with magnetic or electric force densities, which are, in general, functions of space and time.

Electromagnetic fields are defined by forces composed of two parts: those exerted on free charges by electric fields and those exerted on free currents (moving free charges) by magnetic fields. The relative importance of these forces depends on the type of system being considered. In magnetic field systems, as defined in Section 1.1, the important field excitation is provided by the free current density $\mathbf{J}_f$. Hence for magnetic field systems the only important forces arise from the interactions of the free current density $\mathbf{J}_f$ with magnetic fields. Similarly, the only forces of significance in electric field systems, as defined in Section 1.1, are the interactions of free charge density ρ_f with electric fields. The validity of these assumptions is checked in particular problems. Following the pattern established in earlier sections, we treat forces in magnetic field and electric field systems separately. Our object is to describe electromagnetic forces mathematically in alternative forms that will prove useful in work with continuum electromechanical systems.

Two other technically important electromagnetic forces are those resulting from the interactions of polarization density **P** with electric fields and magnetization density **M** with magnetic fields. In Chapters 3 to 5 we calculate total forces on polarizable and magnetizable bodies by using an energy method. We extend this method to account for force densities in polarized or magnetized media that are electrically linear, isotropic, and homogeneous. This limitation in our discussion of polarization and magnetization forces is imposed because use of an energy method requires a knowledge of the mechanical and thermodynamic properties of the material.

8.1 FORCES IN MAGNETIC-FIELD SYSTEMS

Consider first the force resulting from the interaction of moving free charge (i.e., $\mathbf{J}_f$) and a magnetic field. The Lorentz force (1.1.28) gives the total magnetic force on a charge q moving with velocity $\mathbf{v}$ as

$$\mathbf{f} = q\mathbf{v} \times \mathbf{B}. \tag{8.1.1}$$

The force density $\mathbf{F}$ (newtons per cubic meter) can be obtained from this expression by writing

$$\mathbf{F} = \lim_{\delta V \to 0} \frac{\sum_i \mathbf{f}_i}{\delta V} = \lim_{\delta V \to 0} \frac{\sum_i q_i \mathbf{v}_i \times \mathbf{B}_i}{\delta V}, \tag{8.1.2}$$

where $\mathbf{f}_i$, q_i, and $\mathbf{v}_i$ refer to all the particles in δV and $\mathbf{B}_i$ is the flux density experienced by q_i. If we can say that all particles within δV experience the same flux density $\mathbf{B}$, we can use the definition of free current density (see Section B.1.2)* to write (8.1.2) as

$$\mathbf{F} = \mathbf{J}_f \times \mathbf{B}. \tag{8.1.3}$$

The general definition of (8.1.2) requires the averaging of products, whereas the result of (8.1.3) is the product of averages. It is not, in general, true for variables x and y that

$$[xy]_{av} = [x]_{av}[y]_{av}.$$

The force density expressed by (8.1.3) however, agrees, to a high degree of accuracy, with all experimental results obtained with common conductors. The relation (8.1.3) is valid because the volume δV can be made small enough to enclose a region of essentially constant magnetic flux density, although still including many free charges.

In fact, we could have used (8.1.3) rather than (8.1.1) as the definition of **B**, for the original experiments of Biot and Savart and later Ampère† concerned themselves with relating the force density to the free current density

* $\mathbf{J}_f = \lim_{\delta V \to 0} \left[\left(\sum_i q_i \mathbf{v}_i \right) \Big/ \delta V \right]$

† J. D. Jackson, *Classical Electrodynamics*, Wiley, New York 1962, p. 133.