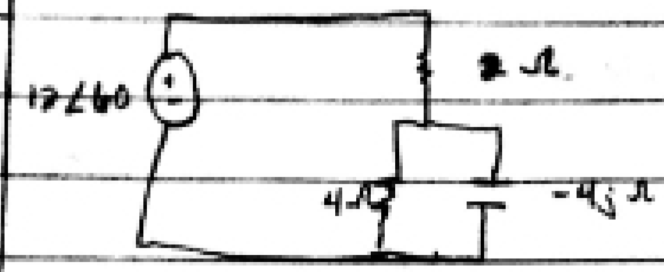
$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$

Resistor $= \frac{V_m I_m}{2} = \frac{V_m^2}{2R} = \frac{I_m^2 R}{2}$

Ex



Find Average Power absorbed by each resistor

For the 2Ω resistor

$Z = 2 + (4 \| -4j\,\Omega)$

$2 + \frac{4(-j4)}{4-j4} \rightarrow 2 + \frac{-j4}{1-j} = 2\angle 0° + \frac{4\angle -90}{\sqrt{2}\angle -45}$

$2 + 2\sqrt{2}\angle 45$

$4 - j2$

$I = \frac{12\angle 60°}{4-j2} = \frac{12\angle 60}{2\sqrt{5}\angle -26.6} = 2.68\angle 86.6°$

$P = \frac{I_m^2 R}{2} = \frac{(2.68)^2\, 2}{2} = 7.2W$

4Ω Resistor

Current division

$I = \left[\frac{-j4}{4-j4}\right](2.68\angle 86.6)$ $\qquad \frac{1\angle -90\,(2.68\angle 86.6)}{\sqrt{2}\angle -45} = 1.9\angle 41.6\,A$

$P = \frac{(1.9)^2\, 4}{2} = 7.21W$

$2.68\angle 86.6 - 1.9\angle 41.6 \quad 1.78\angle 45$

$(0.159 + j2.6751) - (1.42 + j1.26) = I_c = -1.26 + j1.42\,A$

$I_m = 1.9\angle 131.58°$ $\qquad V_c = (1.9\angle 131.57)(4\angle -90) = 7.6\angle 41.58$

$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$ $\qquad P = \frac{(7.6)(1.9)}{2} \cos(41.58 - 131.57)$

Short cut

$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$

RMS voltage or RMS current

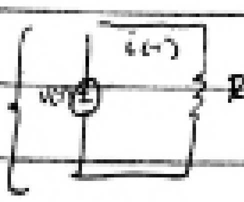
or effective voltage or effective cur

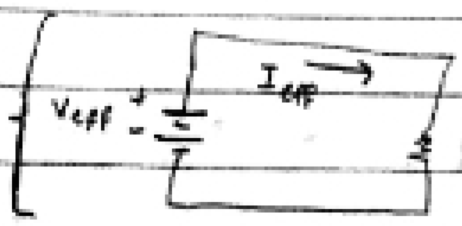
1. The effective value of a periodic current is equal to the DC current that delivers the same average power to a Resistor as the periodic current does

similar defination as current for voltage

2

set up in comparison

a.c  $P = \frac{1}{T} \int_{T_0}^{T_0+T} i^2(t) R \, dt$

D.C  $P = I_{eff}^2 R$

$$I_{eff}^2 R = \frac{1}{T} \int_{T_0}^{T_0+T} i^2(t) R \, dt \rightarrow I_{eff}^2 R = \frac{R}{T} \int_{T_0}^{T_0+T} i^2(t) \, dt$$

$$\sqrt{I_{eff}^2} = \sqrt{\frac{1}{T} \int_{T_0}^{T_0+T} i^2(t) \, dt} \qquad I_{eff} = \sqrt{\frac{1}{T} \int_{T_0}^{T_0+T} i^2(t) \, dt}$$

RMS

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$i^2(t) = I_m^2 \cos^2(\omega t + \theta_i)$$

$$I_{eff} = \sqrt{\frac{1}{T} \int_{T_0}^{T_0+T} I_m^2 \cos^2(\omega t + \theta_i) \, dt} \qquad \cos^2 = \frac{1}{2}(1 + \cos(2\omega t + 2\theta_i))$$

$$\left(\frac{I_m}{2T} \int_{T_0}^{T_0+T} dt + \underbrace{\cancel{\int_{T_0}^{T_0+T} \cos(2\omega t + 2\theta_i)}}_{0} \right)^{1/2}$$

$$I_{eff} = \sqrt{\frac{I_m}{2T} (T) \Big|_{T_0}^{T_0+T}}$$

$$= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = I_{eff}$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$

$$= V_{eff} I_{eff} \cos(\theta_v - \theta_i)$$

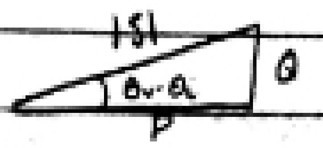
$\frac{V_m}{\sqrt{2}} = V_{eff}$ $\rightarrow$ $v(t) = \sqrt{2} V_{eff} \cos(\omega t + \theta_v)$

Complex Power

A. Definition S = Complex Power

1. $S = P + jQ$
   - S is the Complex Power, Units V.A Volt ampere
   - P is the average (Real) Power, ~~Units~~ watts
   - Q is Reactive Power, Units Volt ampere R or VAR
   - $|S|$ is apparent power, V.A.

2. Power Triangle



$|S| = \sqrt{P^2 + Q^2}$ $(\theta_v - \theta_i) = \tan^{-1} \frac{Q}{P}$

a) $(\theta_v - \theta_i)$ is power factor angle

B) Power Factor, $Pf = \cos(\theta_v - \theta_i)$

$= \frac{P}{|S|}$

Power Factor angle is angle by which the current leads the voltage

$\cos(\theta_v - \theta_i) = \cos(-(\theta_v - \theta_i))$

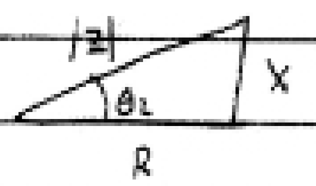
$\theta_v - \theta_i = 26.6°$

$\cos(26.6) = 0.894$ leading and Lagging

$\cos(-26.6) = 0.894$ ELI the ICE man

Lagging leading

Impedance Triangle



$Z = R + jX$ $Z = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle \theta_v - \theta_i$

B. Power Calculations

1. We know that
   a) $P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$
   b) $Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

2. We can write

$S = P + jQ$

$= V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

$c = |c| \angle \theta_c$ $c^* = |c| \angle -\theta_c$

$= V_{rms} I_{rms} (\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i))$

$c = a + jb$ $c^* = a - jb$

$= V_{rms} I_{rms} (e^{j(\theta_v - \theta_i)}$

$*$: complex conjugate

$= V_{rms} I_{rms} \angle \theta_v - \theta_i = (V_{rms} \angle \theta_v)(I_{rms} \angle -\theta_i$

$\rightarrow V_{rms} I^*_{rms} = S = V_{eff} I^*_{eff}$