

General Physics - E&M (PHY 1308) Lecture

Notes

Lecture 011: Resistance to Electric Current

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no tags

Current Density

What if electric current is occurring in a more complex medium (e.g. NOT in a wire)? What about through a cell membrane, which isn't simply describable as a single wire carrying ions into and out of the cell. In that case, we need a more general quantity: current density, the amount of current flowing per unit area of the path.

Current density is by definition a vector, whose direction is given by the direction of current flow at a given point and whose magnitude is the current per unit area. Dividing out microscopic current equation by the area of the conductor:

$$\vec{J} = nq\vec{v}_d.$$

We see the product of drift velocity direction and charge defines the overall direction (n is always positive). So electron current densities point opposite the velocity of electrons, as per our convention that current is in the direction of positive charge motion.

As an exercise, consider an ion pump in a cell membrane. What is an ion pump? It's a protein that is capable of allowing positive ions into or out of the cell itself. That protein is embedded in the cell membrane. We can analyze the current and current density when a channel opens for 1ms and allows 1.1×10^4 singly-ionized potassium ions through ($q = +e$). The radius of the channel, treating it like a little cylinder, is $r = 0.15\text{mm}$.

- What is the current? ANSWER: $I = \Delta Q / \Delta t = 1.8\text{pA}$
- What is the current density? ANSWER: $J = I / A = 2.5 \times 10^7 \text{A/m}^2$

Conduction Mechanisms

The mechanism by which conduction really occurs is beyond the scope of this class, but we can go a long way by using the following model (which is the original model of conduction developed in the 1800s):

- charges are free to move in a conductor and respond to an electric field by moving through the conductor
- conductors are not perfect, however, and the charges suffer collisions with the atoms in the conductor

First, electric fields in conductors? Didn't we use the fact that there are no conductors in electric fields to understand the capacitor problems? Remember, we're not in electrostatic equilibrium here. With the capacitor, we had a specific situation in which charge builds up across an uncrossable gap, and eventually current stops. Here, we are considering a more general situation - when there is current, or when it doesn't stop. In that situation, we are NOT in electrostatic equilibrium and we CAN have electric fields in conductors.

Collisions cause the charges to lose energy they had gained from the electric field. These collisions provide an effective force that works against the electric field force:

$$\vec{F}_{total} = \vec{F}_{electric} + \vec{F}_{collisions}$$

That means that there is a relationship between current density and electric field that is not perfect, but somewhat diminished by the collisions. We can write this as:

$$\vec{J} = \sigma \vec{E}.$$

σ is a property of the material, and is called the conductivity of the material, and ranges between 0 and ∞ in magnitude.

This is a microscopic version of something you might have heard of before:

"Ohm's Law." Ohm's law relates the macroscopic current to the voltage. We can relate the microscopic and macroscopic laws.

First of all, for most common conductors σ is independent of electric field (a constant). Such a material is called "ohmic". Non-ohmic materials have a conductivity that **DEPENDS** on electric field. The microscopic version of Ohm's law is useful in biophysics, geophysics, astrophysics, and electrical engineering, studies that deal routinely with position-dependent electric fields.

- $\sigma = \text{constant}$: Ohmic
- $\sigma(\vec{E})$: non-Ohmic

Conductivity is a measure of how well charges respond to the electric field. A perfect conductor has $\sigma = \infty$, while a perfect insulator has $\sigma = 0$. There is a related quantity, called **resistivity**:

$$\rho = 1/\sigma$$

In terms of resistivity,

$$\vec{J} = \frac{\vec{E}}{\rho}$$

Resistivity tells us how difficult it is for charge to move in a material. The higher the resistivity, the larger the electric field needed to provide the same current density. The macroscopic property of *electrical resistance* is related to resistivity.

The unit of resistivity is the $V \cdot m/A$. One V/A is called an Ohm (Ω), in honor of German Physicist Georg Ohm who explored the relationship between current and voltage. Thus the unit of resistivity is the $\Omega \cdot m$, while units of conductivity are $(\Omega \cdot m)^{-1}$.

Resistivity and conductivity are some of the most variable properties we know about.