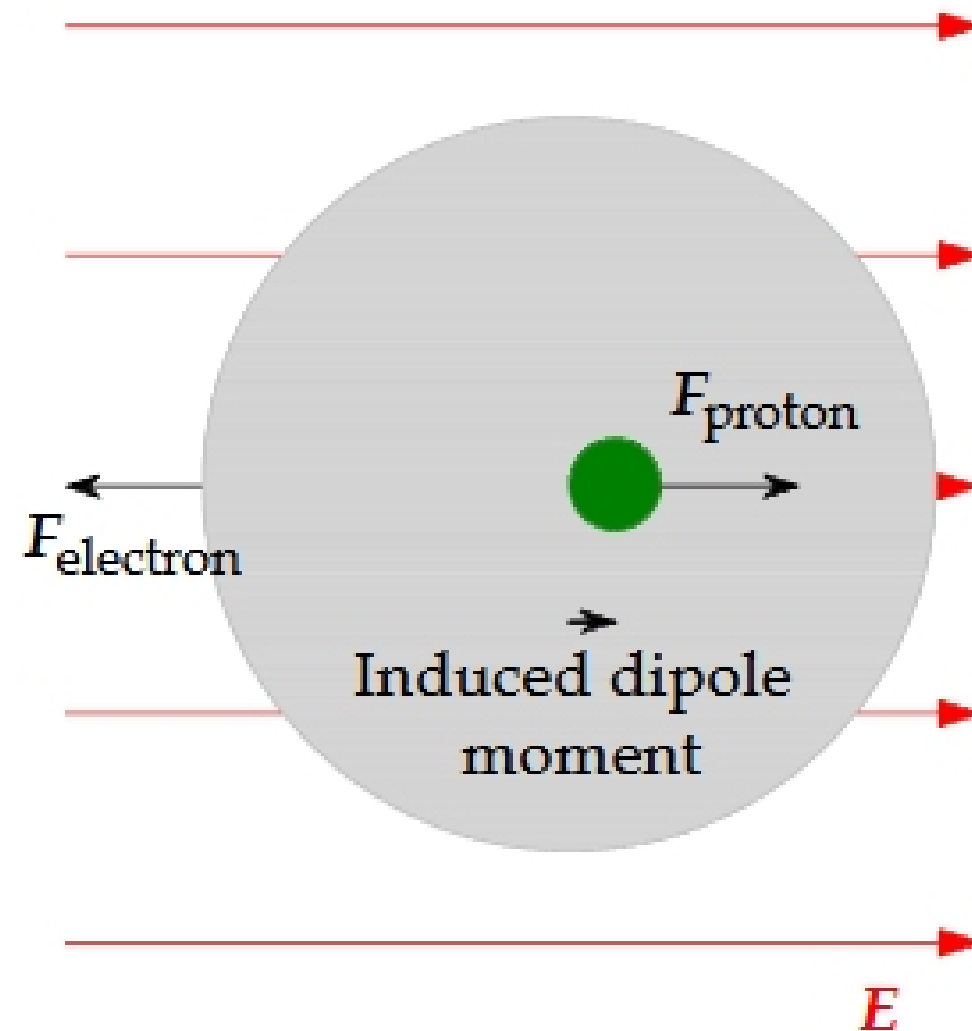

Today in Physics 217: electric dipoles and their interactions

- Origin of coordinates for multipole moments
- Force, torque, and potential energy for dipoles in uniform electric fields
- Force on dipoles in nonuniform electric fields
- Example: dipole *vs.* dipole
- Permanent and induced dipoles in atoms and molecules



Choice of coordinate origin matters a lot in multipole expansion

Changing the origin doesn't change the physics, but it can radically change the terms in the series. Consider a point charge at the origin:

$$V = \frac{q}{r} \quad \text{Monopole only, of course}$$

and one not at the origin, say at $(r, \theta, \phi) = (a, \theta_0, 0)$:

$$V = \frac{q}{r} \left[1 + \frac{a}{r} \cos\theta_0 + \left(\frac{a}{r}\right)^2 P_2(\cos\theta_0) + \left(\frac{a}{r}\right)^3 P_3(\cos\theta_0) + \dots \right]$$

How did a point charge get all these nonzero multipole moments? It didn't, really; the situation still amounts to one point charge, but the accounting is more complicated, when $1/r \neq 1/r$.

Choice of coordinate origin matters a lot in multipole expansion (continued)

- So when someone gives you a charge distribution and asks what all the moments are, they also have to tell you what coordinate system to use.
- One useful exception: if the total charge is zero, then the monopole moment is zero, and the dipole moment is independent of the choice of coordinate origin.

Consider such a situation, and suppose the dipole moment were p originally and p_a in a coordinate system with its origin displaced by some vector a :

$$\begin{aligned} p_a &= \int \mathbf{r}'_a \rho(\mathbf{r}') d\tau' = \int (\mathbf{r}' - \mathbf{a}) \rho(\mathbf{r}') d\tau' \\ &= \int \mathbf{r}' \rho(\mathbf{r}') d\tau' - \mathbf{a} \int \rho(\mathbf{r}') d\tau' = p - \overset{0}{\nearrow} \mathbf{a} = p \quad . \end{aligned}$$