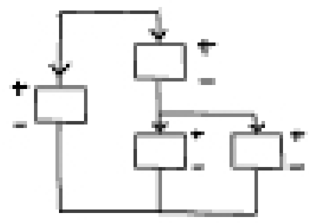
Lecture 5

Circuits

Topics: KVL, KCL, Op=Amps, Thevenin equivalents
Lab Exercises: build robot "head

- Motor servo controller (rotating "neck")
- Phototransistor (robot "eyes)
- Integrate to make a light tracking system

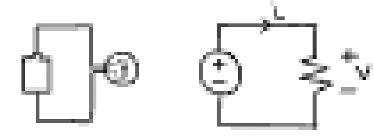


The Circuit Abstraction

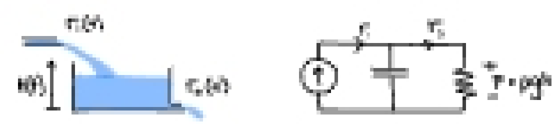
Circuits represent systems as connection of elements

- Through while currents (through variables) flow
- Across while voltages (across variables) develop

We can represent the flashlight as a voltage source, a battery, connected to a resistor, a light bulb
The voltage source generates a voltage v across the resistor and a current i through the resistor



We can represent the flow of water by a circuit
Flow of water into and out of tank are represented as "thought variables" r and r respectively
Hydraulic pressure at bottom of tank is represented by the "across variable" $P = \rho gh$



Circuits are important for two very different reasons:

- As a physical system
  - Power—from generators and transformers to power lines
  - Electronics—from cell phones to computers
- As models of complex systems
  - Neurons
  - Brain
  - Cardiovascular system
  - Hearing

Circuits are the basis of the enormously successful semiconductor industry
Circuits as models of complex systems: myelinated neutron

The primitives are the elements:

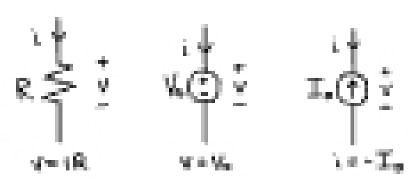
- Sources, capacitors, resistors

The rules of combination are the rules that govern

- Flow of current (through variable
- Development of voltage

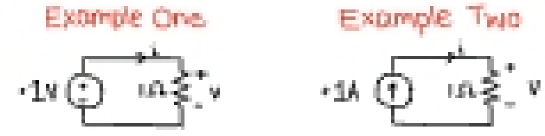
Analyzing Circuits

We will start with the simplest elements: resistors and sources



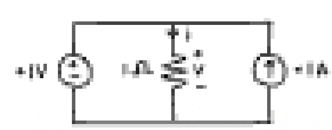
Analyzing simple circuits is straightforward

Example 1: The voltage source determines the voltage across the resistor, $v = 1V$, so the current through the resistor is $i = v/R = 1A$
Example 2: The current course determines the current through the resistor, $i = 1A$, so the voltage across the resistor is $v = iR = 1V$



Check yourself: Analyzing simple circuits

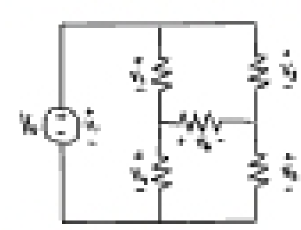
Correct answer: 1
The current through the resistor is 1A



More complex circuits can be analyzed by systematically applying Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL)

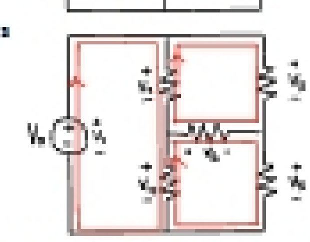
Analyzing Circuits: KVL

The sum of the voltages around any closed path is zero
Example: $-v_1 + v_2 + v_4 = 0$



Check yourself: Kirchhoff's Voltage Law

Correct answer: 3
How many KVL relations are there for this circuit ? There are 7 loops



Planar circuits can be characterized by their inner loops
KVL equations for the inner loops are independent

A: $-v_1 + v_2 + v_4 = 0$
B: $-v_2 + v_3 - v_5 = 0$
C: $-v_4 + v_5 + v_6 = 0$

All possible KVL equations for planar circuits can be generated by combinations of the inner loops

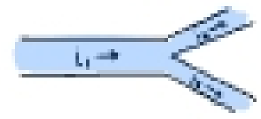
A + B = 0

One KVL equation can be written for every closed path in a circuit
Sets of KVL equations are not necessarily linearly independent

KVL equations for the inner loops of planar circuits are linearly independent

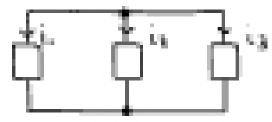
Analyzing Circuits: KCL

The flow of electrical current is analogous to the flow of incompressible fluid, such as water



Current i_1 flows into a node and two currents i_2 and i_3 flow out

$i_1 = i_2 + i_3$



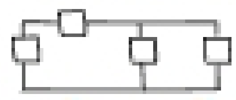
The net flow of electrical current into or out of a node is zero

The net current out of the top node must be zero

$i_1 + i_2 + i_3 = 0$

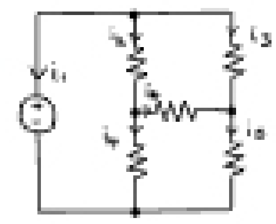
Electrical currents cannot accumulate in elements, so current that flows into a circuit element also flows out

Check yourself: Kirchhoff's Current Law



Correct answer: 2

There are three equations that can be written but only two of them are linearly independent KCL equations



Check yourself: Kirchhoff's Current Law

Correct answer: 1

There are three linearly independent KCL equations

General form: the number of linearly independent KCL equation is the number of nodes minus one

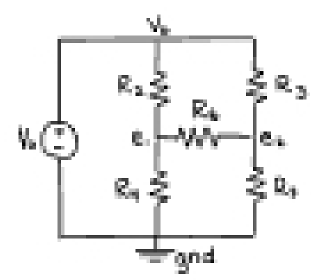
The net current out of any closed surface (which can contain multiple nodes) is zero

KVL, KCL, and Constitutive Equations

Circuits can be analyzed by combining

- All linearly independent KVL equations
- All linearly independent KCL equations
- One constitutive equation for each element

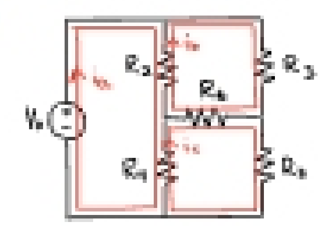
Node Voltages



The "node" method is one of many ways to systematically reduce the number of circuit equations and unknowns

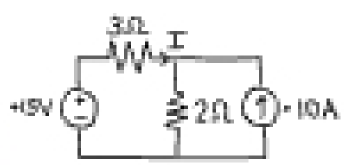
- Label all nodes except one: ground = 0 V
- Write KCL for each node whose voltage is not unknown
- Solve for the unknowns

Loop Currents



The "loop current" method is another way to systematically reduce the number of circuit equations and unknowns

- Label all the loop currents
- Write KVL for each loop
- Solve for the unknowns



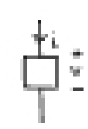
Check yourself: Current

Correct answer: 3

The current in the circuit is -1A

Common Patterns

Circuits can be simplified when two or more elements behave as a single element



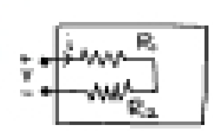
A "one-port" is a circuit that can be represented as a single element

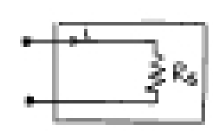
A one-port has two terminals

Current enters one terminal (+) and exits another (-), producing a voltage (v) across the terminals

Series Combinations

The series combination of two resistors is equivalent to a single resistor whose resistance is the sum of the two original resistances: $R_S = R_1 + R_2$

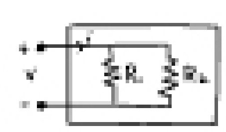


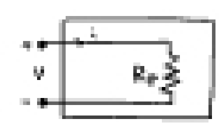


The resistance of a series of combination is always larger than either of the original resistances

Parallel Combinations

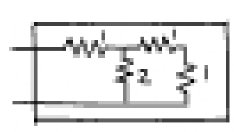
The parallel combination of two resistors is equivalent to a single resistor whose conductance (1/resistance) is the sum of the two original conductances





The resistance of a parallel combination is always smaller than either of the original resistances

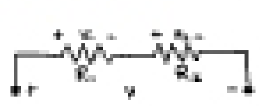
Check yourself: Equivalent Resistance



Correct answer: 3

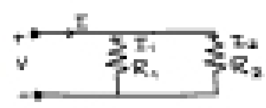
The equivalent resistance of the one-port is 2 ohms

Voltage Divider



Resistors in series act as voltage dividers

Current Divider



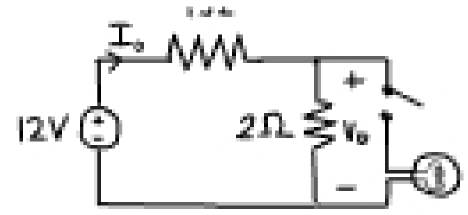
Resistors in parallel act as current dividers

Interaction of Circuit Elements

Circuit design is complicated by interactions among the elements

Adding an element change voltages and currents throughout the circuit

Example: closing a switch is equivalent to adding a new element



Check yourself: Closing a switch

Correct answer: 2

Closing the switch cause V_0 to decrease and I_0 to increase

One Ports:

If a circuit connects to the world via just two terminals, then that circuit can be represented by a single generalized element called a one-port, regardless of how many components are in the circuit

This is analogous to

- replacing delays, gains and adders with a system function
- combining a sequence of operations in a procedure call
- combining diverse data in a list

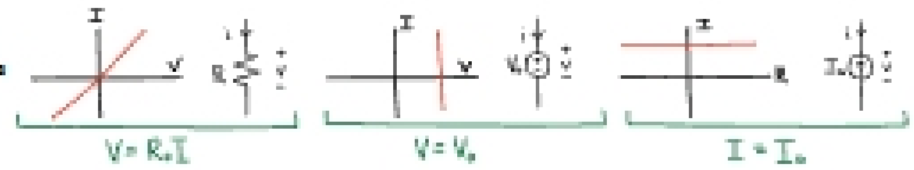
These representations are compositional—they replace multiple elements with a single element that can be used in the same way that primitives are used

Current-Voltage Relations

Current-voltage relations for resistors and sources are straight lines

Current-voltage relations are analogous to system functionals

They summarize the behavior of a one-port



Parallel One-Ports

If the i-v curves for two one-ports are both straight lines, then the i-v curve for the parallel combination is a straight line

This is because the sum of two straight lines is a straight line

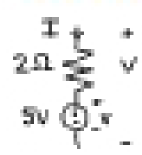
Series One-Ports

If the i-v curves for two one-ports are both straight lines, then the i-v curve for the series combination is a straight line

This is because the horizontal sum of two straight lines is a straight line

Check yourself: Current-Voltage

Correct answer: 1



Summary

Circuits represent systems as connection of elements

- Through which currents (through variables) flow
- Across while voltages (across variables) develop

We have seen three (of many) methods for analyzing circuits, each one bases on a different set of variables

- currents and voltages for each element
- node voltages
- loop currents

We can simplify analysis by recognizing common patters

- series and parallel combinations
- voltage and current dividers

We have seen our first major abstraction: the one-port and a representation to characterize it—the current-voltage relation