Chapter 7. Electrodynamics

7.1. Electromotive Force

An electric current is flowing when the electric charges are in motion. In order to sustain an electric current we have to apply a force on these charges. In most materials the current density $\bar{J}$ is proportional to the force per unit charge:

$$\bar{J} = \sigma \bar{f}$$

The constant of proportionality σ is called the **conductivity** of the material. Instead of specifying the conductivity, it is more common to specify the **resistivity** ρ:

$$\rho = \frac{1}{\sigma}$$

For conductors the resistivity is typically 10^{-8} Ω-m; for semiconductor it varies between 0.01 Ω-m and 1 Ω-m, and for insulators it varies between 10^5 Ω-m and 10^6 Ω-m. In most cases the force on the charges is the electromagnetic force. In that case the current density is equal to:

$$\bar{J} = \sigma\left(\bar{E} + \bar{v} \times \bar{B}\right)$$

If the velocity of the charges is small the second term can be ignored, and the equation for $\vec{J}$ reduces to **Ohm's Law**:

$$\bar{J} = \sigma \bar{E}$$

Consider a wire of cross-sectional area A and length L. If a potential difference V is applied between the ends of the wire, it will produce an electric field inside the wire of magnitude

$$E = \frac{V}{L}$$

The current density in the wire is therefore equal to

$$J = \sigma \frac{V}{L}$$

The total current flowing through the wire is therefore equal to

$$I = JA = \sigma A \frac{V}{L}$$

This equation shows that the current flowing from one electrode to the other electrode is proportional to the potential difference between them. This is a rather surprising result since the charge carriers are constantly accelerating. However, the proportionality between the current and the potential difference has been found to be correct for most materials. This relation can be written as

$$V = IR$$

The constant of proportionality R is called the **resistance** of the material. It is in general a function of the geometry of the system and the conductivity of the materials between the electrodes. The unit of resistance is the **ohm** (Ω). The resistance of the wire is equal to

$$R = \frac{V}{I} = \frac{V}{\sigma A \frac{V}{L}} = \frac{1}{\sigma}\frac{L}{A} = \rho \frac{L}{A}$$

To create a current we have to do work. The work required to move a unit of charge across a potential difference V is equal to V. To establish a current I, we need to deliver a power P where

$$P = VI = I^2 R$$

The unit of power is the **Watt** (1 W = 1 J/s). The work done by the electric force on the charge carriers is converted into heat (**Joule heating**).

Example: Problem 7.1

Two concentric metal spherical shells, of radius a and b, respectively, are separated by weakly conducting material of conductivity σ.

a) If they are maintained at a potential difference V, what current flows from one to the other?
b) What is the resistance between the shells?

a) Suppose a charge Q is placed on the inner shell. The electric field in the region between the shells will be equal to

$$\overline{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

The corresponding potential difference between the spheres is equal to

$$V_a - V_b = -\int_b^a \overline{E} \bullet d\overline{r} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Therefore, in order to maintain a potential difference V between the spheres, we must place a charge Q equal to

$$Q = \frac{4\pi\varepsilon_0 V}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

on the center shell. The total current flowing between the two shells is equal to

$$I = \oint_{Sphere} \overline{J} \bullet d\overline{a} = \sigma \oint_{Sphere} \overline{E} \bullet d\overline{a} = \sigma \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} 4\pi r^2 = \sigma \frac{Q}{\varepsilon_0} = 4\pi\sigma \frac{V}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

b) The resistance between the shells can be obtained from Ohm's law:

$$R = \frac{V}{I} = \frac{V}{4\pi\sigma \frac{V}{\left(\frac{1}{a} - \frac{1}{b}\right)}} = \frac{1}{4\pi\sigma}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Example: Problem 7.2

a) Two metal objects are embedded in weakly conducting material of conductivity σ (see Figure 7.1). Show that the resistance between them is related to the capacitance of the arrangement by

$$R = \frac{\varepsilon_0}{\sigma C}$$

b) Suppose you connected a battery between 1 and 2 and charged them up to a potential difference V_0. If you then disconnect the battery, the charge will gradually leak off. Show that $V(t) = V_0 \exp(-t/\tau)$, and find the time constant τ in terms of ε_0 and σ.

a) Suppose a charge Q is placed on the positively charged conductor. The current flowing from the positively charged conductor is equal to