PROBLEMS

Section 11.2—Transmission Line Parameters

11.1 An air-filled planar line with $w = 30$ cm, $d = 1.2$ cm, $t = 3$ mm has conducting plates with $\sigma_c = 7 \times 10^7$ S/m. Calculate R, L, C, and G at 500 MHz.

11.2 A coaxial cable has an inner conductor of radius $a = 0.8$ mm and an outer conductor of radius $b = 2.6$ mm. The conductors have $\sigma_c = 5.28 \times 10^7$ S/m, $\mu_c = \mu_o$, and $\epsilon_c = \epsilon_o$; they are separated by a dielectric material having $\sigma = 10^{-5}$ S/m, $\mu = \mu_o$, $\epsilon = 3.5\,\epsilon_o$. At 80 MHz, calculate the line parameters L, C, G, and R.

11.3 The copper leads of a diode are 16 mm in length and have a radius of 0.3 mm. They are separated by a distance of 2 mm as shown in Figure 11.44. Find the capacitance between the leads and the ac resistance at 10 MHz.

Section 11.3—Transmission Line Equations

*11.4 In Section 11.3, it was mentioned that the equivalent circuit of Figure 11.5 is not the only possible one. Show that eqs. (11.4) and (11.6) would remain the same if the Π-type and T-type equivalent circuits shown in Figure 11.45 were used.

FIGURE 11.44 The diode of Problem 11.3.

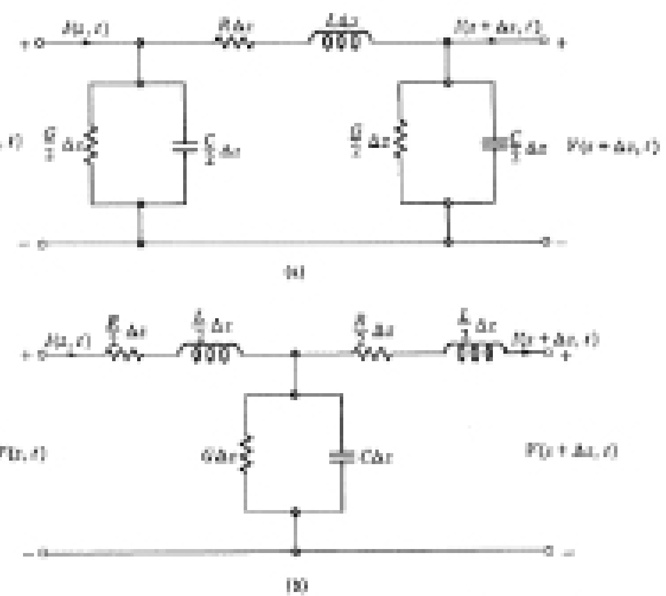

FIGURE 11.45 Equivalent circuits for Problem 11.4: (a) Π-type, (b) T-type.

11.5 (a) Show that at high frequencies ($R \ll \omega L$, $G \ll \omega C$),

$$\gamma \simeq \left(\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}$$

(b) Obtain a similar formula for Z_o.

11.6 A lossless cable has parameters $L = 0.42$ μH/m and $C = 80$ pF/m. Find the velocity and the characteristic impedance of the cable.

11.7 At 60 MHz, the following characteristics of a lossy line are measured:

$$Z_o = 50\Omega, \quad \alpha = 0.04 \text{ dB/m}, \quad \beta = 2.5 \text{ rad/m}$$

Calculate R, L, C, and G of the line.

11.8 A 78 Ω lossless planar line was designed but did not meet a requirement. What fraction of the widths of the strip should be added or removed to get the characteristic impedance of 75 Ω?

11.9 A telephone line operating at 1 kHz has $R = 6.8$ Ω/mi, $L = 3.4$ mH/mi, $C = 8.4$ nF/mi, and $G = 0.42$ μS/mi.
Find (a) Z_o and γ, (b) phase velocity, (c) wavelength.

11.10 A lossless transmission line has $L = 6.5$ μH/m and $C = 40$ nF/m. Calculate the velocity ratio u/c for this case.

11.11 A distortionless line operating at 120 MHz has $R = 20$ Ω/m, $L = 0.3$ μH/m, and $C = 63$ pF/m. (a) Determine γ, u, and Z_o. (b) How far will a voltage wave travel before it is reduced to 20% of its initial magnitude? (c) How far will it travel to suffer a 45° phase shift?

11.12 A coaxial cable has its conductors made of copper ($\sigma_c = 5.8 \times 10^7$ S/m) and its dielectric made of polyethylene ($\epsilon_r = 2.25$, $\mu_r = 1$). If the radius of the outer conductor is 3 mm, determine the radius of the inner conductor so that $Z_o = 75\Omega$.

11.13 For a lossless two-wire transmission line, show that

(a) The phase velocity $u = c = \dfrac{1}{\sqrt{LC}}$

(b) The characteristic impedance $Z_o = \dfrac{120}{\sqrt{\epsilon_r}}\cosh^{-1}\dfrac{d}{2a}$

Is part (a) true of other lossless lines?

11.14 A twisted line, which may be approximated by a two-wire line, is very useful in the telephone industry. Consider a line comprising two copper wires of diameter 0.12 cm that have a 0.32 cm center-to-center spacing. If the wires are separated by a dielectric material with $\epsilon = 3.5\epsilon_o$, find L, C, and Z_o.

11.15 On a distortionless line, the voltage wave is given by

$$V(\ell') = 120e^{0.0025\ell'}\cos(10^8 t + 2\ell') + 60e^{-0.0025\ell'}\cos(10^8 t - 2\ell')$$

where ℓ' is the distance from the load. If $Z_L = 300\ \Omega$, find (a) α, β, and u, (b) Z_o and $I(\ell')$.

11.16 The voltage on a line is given by

$$V(\ell) = 80e^{10^{-3}\ell}\cos(2\pi \times 10^8 t + 0.01\ell) + 60e^{-10^{-3}\ell}\cos(2\pi \times 10^8 t + 0.01\ell)\ \text{V}$$

where ℓ is the distance from the load. Calculate γ and s.

11.17 A distortionless transmission line satisfies $RC = LG$. If the line has $R = 10$ mΩ/m, $C = 82$ pF/m, and $L = 0.6$ μH/m, calculate its characteristic impedance and propagation constant. Assume that the line operates at 80 MHz.

11.18 A coaxial line 5.6 m long has distributed parameters $R = 6.5$ Ω/m, $L = 3.4$ μH/m, $G = 8.4$ mS/m, and $C = 21.5$ pF/m. If the line operates at 2 MHz, calculate the characteristic impedance and the end-to-end propagation time delay.

Section 11.4—Input Impedance, Standing Wave Ratio, and Power

11.19 (a) Show that a transmission coefficient may be defined as

$$\tau_L = \frac{V_L}{V_o^+} = 1 + \Gamma_L = \frac{2Z_L}{Z_L + Z_o}$$

(b) Find τ_L when the line is terminated by (i) a load whose value is nZ_o, (ii) an open circuit, (iii) a short circuit, (iv) $Z_L = Z_o$ (matched line).

11.20 A standing wave has a minimum field of 20 μV/m and a maximum field of 120 μV/m. Calculate (a) the standing wave ratio, (b) the reflection coefficient.

11.21 A lossy transmission line has $R = 3.5$ Ω/m, $L = 2$ μH/m, $C = 120$ pF/m, and $G = 0$. At 400 MHz, determine α, β, Z_o, and u.

11.22 Show that a lossy transmission line of length ℓ has an input impedance $Z_{sc} = Z_o \tanh \gamma\ell$ when shorted and $Z_{oc} = Z_o \coth \gamma\ell$ when open. Confirm eqs. (11.41) and (11.42).

11.23 Find the input impedance of a short-circuited coaxial transmission line of Figure 11.46 if $Z_o = 65 + j38$ Ω, $\gamma = 0.7 + j2.5$/m, $\ell = 0.8$ m.

11.24 A 50-Ω transmission line of length ℓ is open-circuited. If the input impedance is $-j62$ Ω, determine ℓ in terms of λ.

11.25 Refer to the lossless transmission line shown in Figure 11.47. (a) Find Γ and s. (b) Determine Z_{in} at the generator.

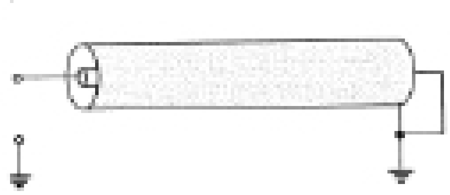

FIGURE 11.46 For Problem 11.23.

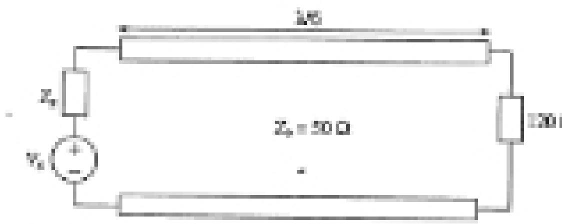

FIGURE 11.47 For Problem 11.25.

11.26 A 60 Ω lossless line is connected to a source with $V_g = 10\angle 0°\ V_{rms}$ and $Z_g = 50 - j40$ Ω and terminated with a load of $j40$ Ω. If the line is 100 m long and $\beta = 0.25$ rad/m, calculate Z_{in} and V at

(a) The sending end

(b) The receiving end

(c) 4 m from the load

(d) 3 m from the source

11.27 A lossless transmission line with a characteristic impedance of 75 Ω is terminated by a load of 120 Ω. The length of the line is 1.25λ. If the line is energized by a source of 100 V (rms) with an internal impedance of 50 Ω, determine (a) the input impedance and (b) the magnitude of the load voltage.

*11.28 Consider the two-port network shown in Figure 11.48(a). The relation between the input and output variables can be written in matrix form as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

For the lossy line in Figure 11.48(b), show that the $ABCD$ matrix is

$$\begin{bmatrix} \cosh \gamma\ell & Z_o \sinh \gamma\ell \\ \frac{1}{Z_o}\sinh \gamma\ell & \cosh \gamma\ell \end{bmatrix}$$

11.29 A 100-Ω line is terminated in a load. If the reflection coefficient is $\Gamma_L = 0.66 + j0.15$, calculate (a) the load impedance, (b) the standing wave ratio.

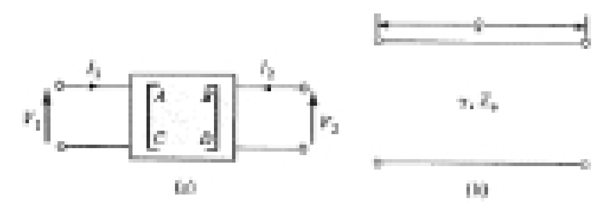

FIGURE 11.48 For Problem 11.28: (a) network, (b) lossy line.

Section 11.5—The Smith Chart

11.30 A quarter-wave lossless 100 Ω line is terminated by a load $Z_L = 210\ \Omega$. If the voltage at the receiving end is 80 V, what is the voltage at the sending end?

11.31 A 50 Ω transmission line is terminated by a $100 + j150\ \Omega$ load. How far from the load will the line impedance be $50 + j110\ \Omega$?

11.32 Two lines are cascaded as shown in Figure 11.49. Determine:

(a) The input impedance

(b) The standing wave ratio for sections XY and YZ

(c) The reflection coefficient at Z

11.33 A 50 Ω lossless line is 4.2 m long. At the operating frequency of 300 MHz, the input impedance at the middle of the line is $80 - j60\ \Omega$. Find the input impedance at the generator and the voltage reflection coefficient at the load. Take $u = 0.8c$.

11.34 A lossless transmission line, with characteristic impedance of 50 Ω and electrical length of $\ell = 0.27\lambda$, is terminated by a load impedance $40 - j25\ \Omega$. Determine Γ_L, s, and Z_{in}.

11.35 A 75-Ω lossless transmission line is 132 cm long, with a dielectric constant $\varepsilon_r = 3.42$. If the line operates at 400 MHz with an input impedance of $Z_{in} = 40 + j65\ \Omega$, use the Smith chart to determine the terminating load.

11.36 The distance from the load to the first minimum voltage in a 50-Ω line is 0.12λ and the standing wave ratio $s = 4$.

(a) Find the load impedance Z_L.

(b) Is the load inductive or capacitive?

(c) How far from the load is the first maximum voltage?

11.37 A lossless 50 Ω line is terminated by a load $Z_L = 75 + j60\ \Omega$. Using a Smith chart, determine (a) the reflection coefficient Γ, (b) the standing wave ratio s, (c) the input impedance at 0.2λ from the load, (d) the location of the first minimum voltage from the load, (e) the shortest distance from the load at which the input impedance is purely resistive.

11.38 A transmission line is terminated by a load with admittance $Y_L = (0.6 + j0.8)/Z_o$. Find the normalized input impedance at $\lambda/8$ from the load.

11.39 For a transmission line, $f = 100$ MHz, $u = 2 \times 10^8$ m/s, $\ell = 1.2$ m, and $Z_o = 50\ \Omega$. If the line is terminated in a load $Z_L = 150 + j200\ \Omega$, determine the input impedance.

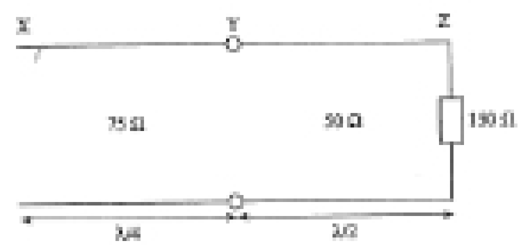

FIGURE 11.49 For Problem 11.32.

11.40 An 80 Ω transmission line operating at 12 MHz is terminated by a load Z_L. At 22 m from the load, the input impedance is $100 - j120\ \Omega$. If $u = 0.8c$,

(a) Calculate Γ_L, $Z_{in,max}$, and $Z_{in,min}$.

(b) Find Z_L, s, and the input impedance at 28 m from the load.

(c) How many $Z_{in,max}$ and $Z_{in,min}$ are there between the load and the $100 - j120\ \Omega$ input impedance?

11.41 An antenna, connected to a 150 Ω lossless line, produces a standing wave ratio of 2.6. If measurements indicate that voltage maxima are 120 cm apart and that the last maximum is 40 cm from the antenna, calculate

(a) The operating frequency

(b) The antenna impedance

(c) The reflection coefficient (assume that $u = c$).

11.42 An 80 Ω lossless line has $Z_L = j60\ \Omega$ and $Z_{in} = j40\ \Omega$. (a) Determine the shortest length of the line. (b) Calculate s and Γ_L.

11.43 A 75 Ω lossless line is terminated by an unknown load impedance Z_L. If at a distance 0.2λ from the load the voltage is $V_s = 2 + j$ V while the current is 10 mA, find Z_L and s.

11.44 Two $\lambda/4$ transformers in tandem are to connect a 50 Ω line to a 75 Ω load as in Figure 11.50.

(a) Determine the characteristic impedance Z_{o1} if $Z_{o2} = 30\ \Omega$ and there is no reflected wave to the left of A.

(b) If the best results are obtained when

$$\left[\frac{Z_o}{Z_{o1}}\right]^2 = \frac{Z_{o1}}{Z_{o2}} = \left[\frac{Z_{o2}}{Z_L}\right]^2$$

determine Z_{o1} and Z_{o2} for this case.

11.45 Two identical antennas, each with input impedance 74 Ω, are fed with three identical 50 Ω quarter-wave lossless transmission lines as shown in Figure 11.51. Calculate the input impedance at the source end.

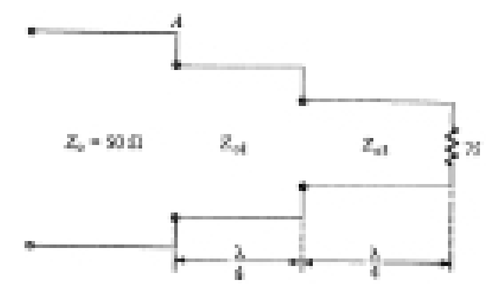

FIGURE 11.50 Double section transformer of Problem 11.44.