Chapter 2. Electrostatics

2.1. The Electrostatic Field

To calculate the force exerted by some electric charges, $q_1, q_2, q_3, \ldots$ (**the source charges**) on another charge Q (**the test charge**) we can use the **principle of superposition**. This principle states that the interaction between any two charges is completely unaffected by the presence of other charges. The force exerted on Q by q_1, q_2, and q_3 (see Figure 2.1) is therefore equal to the vector sum of the force $\overline{F}_1$ exerted by q_1 on Q, the force $\overline{F}_2$ exerted by q_2 on Q, and the force $\overline{F}_3$ exerted by q_3 on Q.

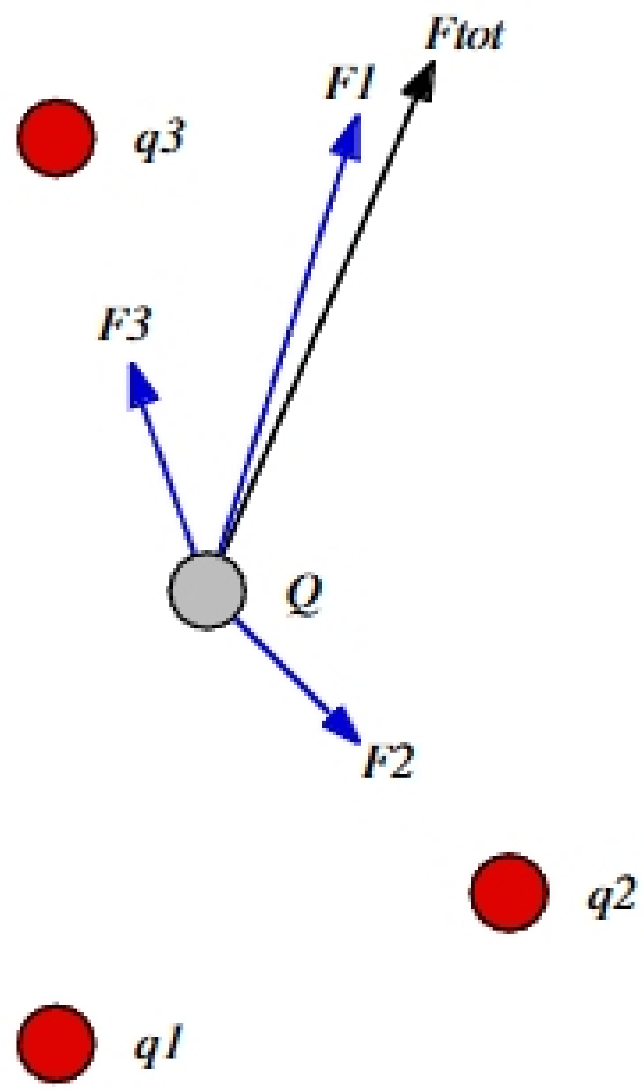

Figure 2.1. Superposition of forces.

The force exerted by a charged particle on another charged particle depends on their separation distance, on their velocities and on their accelerations. In this Chapter we will consider the special case in which the source charges are stationary.

The **electric field** produced by stationary source charges is called and **electrostatic field**. The electric field at a particular point is a vector whose magnitude is proportional to the total force acting on a test charge located at that point, and whose direction is equal to the direction of the force acting on a positive test charge. The electric field $\overline{E}$, generated by a collection of source charges, is defined as

$$\overline{E} = \frac{\overline{F}}{Q}$$

where $\overline{F}$ is the total electric force exerted by the source charges on the test charge Q. It is assumed that the test charge Q is small and therefore does not change the distribution of the source charges. The total force exerted by the source charges on the test charge is equal to

$$\overline{F} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \ldots = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{r_1^2}\hat{r}_1 + \frac{q_2 Q}{r_2^2}\hat{r}_2 + \frac{q_3 Q}{r_3^2}\hat{r}_3 + \ldots\right) = \frac{Q}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

The electric field generated by the source charges is thus equal to

$$\overline{E} = \frac{\overline{F}}{Q} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

In most applications the source charges are not discrete, but are distributed continuously over some region. The following three different distributions will be used in this course:

1. **line charge** λ: the charge per unit length.

2. **surface charge** σ: the charge per unit area.

3. **volume charge** ρ: the charge per unit volume.

To calculate the electric field at a point $\overline{P}$ generated by these charge distributions we have to replace the summation over the discrete charges with an integration over the continuous charge distribution:

1. for a line charge: $\overline{E}(\overline{P}) = \frac{1}{4\pi\varepsilon_0}\int_{Line}\frac{\hat{r}}{r^2}\lambda dl$

2. for a surface charge: $\overline{E}(\overline{P}) = \frac{1}{4\pi\varepsilon_0}\int_{Surface}\frac{\hat{r}}{r^2}\sigma da$

3. for a volume charge: $\overline{E}(\overline{P}) = \frac{1}{4\pi\varepsilon_0}\int_{Volume}\frac{\hat{r}}{r^2}\rho d\tau$

Here $\hat{r}$ is the unit vector from a segment of the charge distribution to the point $\overline{P}$ at which we are evaluating the electric field, and r is the distance between this segment and point $\overline{P}$.

Example: Problem 2.2

a) Find the electric field (magnitude and direction) a distance z above the midpoint between two equal charges q a distance d apart. Check that your result is consistent with what you would expect when $z \gg d$.

b) Repeat part a), only this time make he right-hand charge $-q$ instead of $+q$.

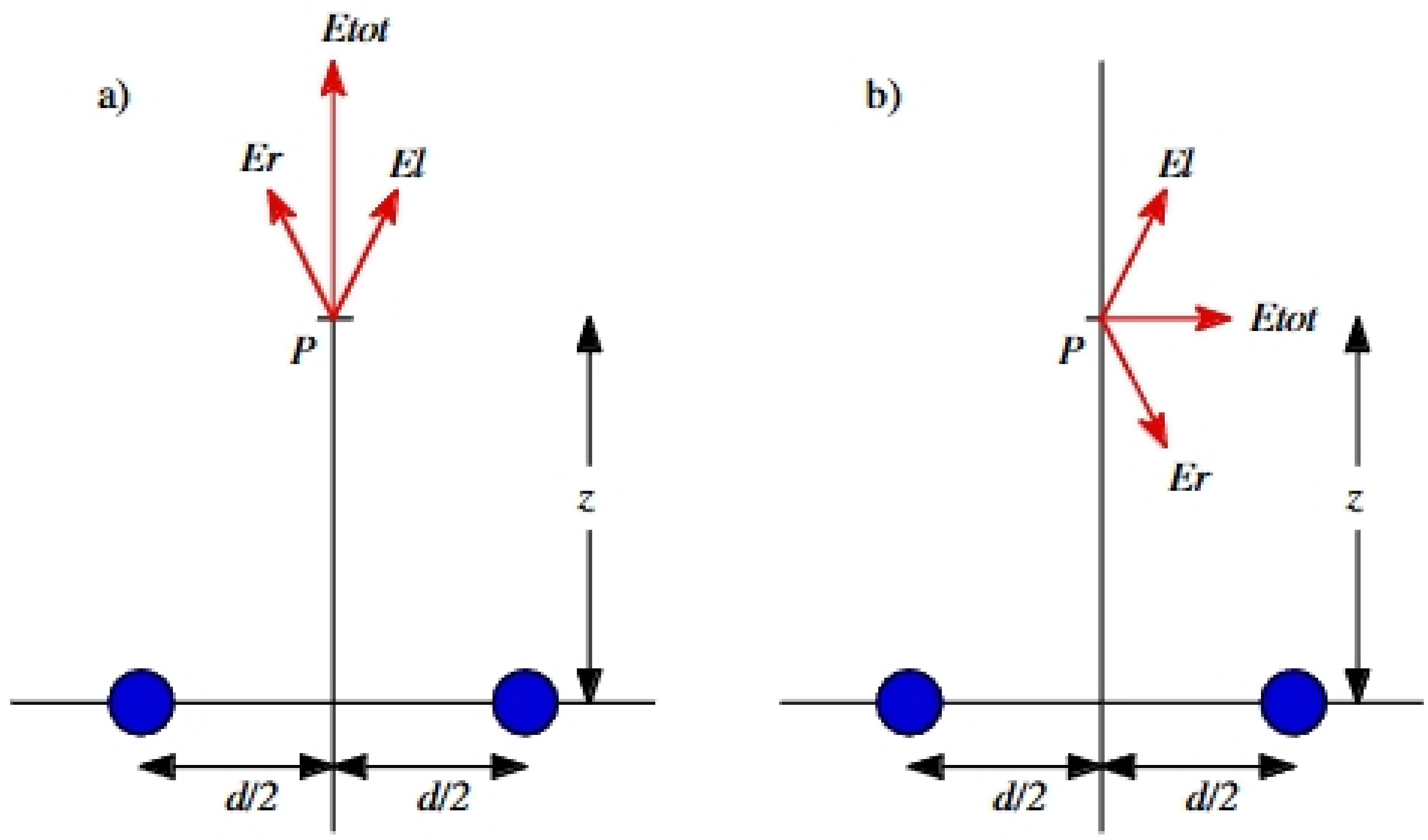

Figure 2.2. Problem 2.2

a) Figure 2.2a shows that the x components of the electric fields generated by the two point charges cancel. The total electric field at P is equal to the sum of the z components of the electric fields generated by the two point charges:

$$\overline{E}(\overline{P}) = 2\frac{1}{4\pi\varepsilon_0}\frac{q}{\left(\frac{1}{4}d^2+z^2\right)}\frac{z}{\sqrt{\frac{1}{4}d^2+z^2}}\hat{z} = \frac{1}{2\pi\varepsilon_0}\frac{qz}{\left(\frac{1}{4}d^2+z^2\right)^{3/2}}\hat{z}$$

When $z \gg d$ this equation becomes approximately equal to

$$\overline{E}(\overline{P}) \cong \frac{1}{2\pi\varepsilon_0}\frac{q}{z^2}\hat{z} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{z^2}\hat{z}$$