$y' - \frac{1}{x} y = -\frac{2}{x}$ $\quad y(1) = 2$

I.F. $= e^{-\int \frac{1}{x}} = \frac{1}{x}$

$\frac{1}{x} y' - \left(\frac{1}{x}\right)\left(\frac{1}{x}\right) y = \left(\frac{-2}{x}\right)\left(\frac{1}{x}\right)$

$\frac{1}{x} y' - \frac{1}{x^2} y = \frac{-2}{x^2}$

$\int \left(\frac{1}{x} y\right)' = \int \frac{-2}{x^2} = 2/x$

$\frac{1}{x} y = \frac{2}{x} + C$

$y = 2 + xC$ $\quad C = 0$

$y = 2$ $\quad$ IR H

5-19-14 $\quad$ 2.1 $\quad$ (7) $\quad y' + 2ty = 2te^{-t^2}$ $\quad y' = -2ty + \frac{2t}{e^{t^2}}$ $\quad 2ty = \frac{2t}{e^{t^2}}$ $\quad y = \frac{1}{e^{t^2}}$

$y' = -2ty + \frac{2t}{e^{t^2}}$ $\quad$ conversant

$y' + 2ty = 2te^{-t^2}$ $\quad e^{t^2} y' + 2te^{t^2} y = 2t$ $\quad (e^{t^2} y)' = 2t$

$e^{t^2} y = t^2 + C$

$y = \frac{t^2}{e^{t^2}} + \frac{C}{e^{t^2}}$

Separable Equations

Form of it $\quad M(x) + N(y) \frac{dy}{dx} = 0 \quad$ or $\quad M(x)\,dx + N(y)\,dy = 0$

Example

$\frac{dy}{dx} = \frac{3x^2}{y^2 - 1}$ $\quad (y^2 - 1)\frac{dy}{dx} = 3x^2 \rightarrow -3x^2 + (y^2 - 1)\frac{dy}{dx} = 0 \rightarrow \frac{d(-x^3)}{dx} + \frac{d\left(\frac{y^3}{3} - y\right)}{dy}$

or $\int (y^2 - 1)\,dy = \int 3x^2\,dx$

$\frac{y^3}{3} - y = x^3 + C$ $\leftarrow$ Implicit solution

$\frac{dy}{dx} = \frac{y^3}{x^4}$ $\quad \int \frac{dy}{y^3} = \int \frac{dx}{x^4}$ $\quad \ln(y^3) = \ln x^4$

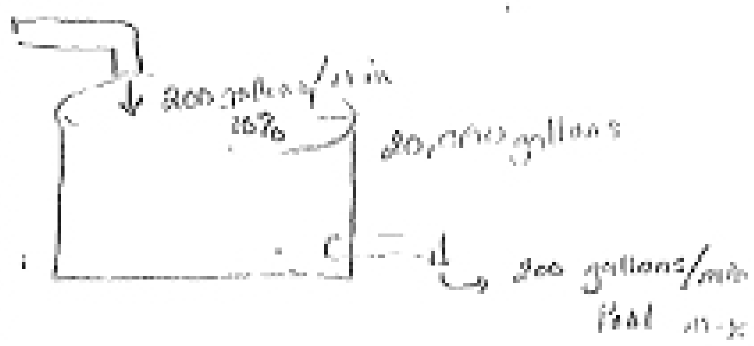
or

$\frac{dy}{dx} = \frac{x^{-4}}{y^{-3}}$ $\quad \int y^{-3}\,dy = \int x^{-4}\,dx$ $\quad \frac{-y^{-2}}{2} = \frac{-x^{-3}}{3} + C$

$\frac{1}{2y^2} = \frac{1}{3x^3} - C$ $\quad \frac{1}{2y^2} = \frac{1 - 3x^3 C}{3x^3}$ $\quad y^2 = \frac{3x^3}{2 - 6x^3 C}$

$y(1) = 3$

$y = \pm\sqrt{\frac{3x^3}{2 - 6x^3 C}}$



Write a differential equation for a $Q(t)$ which is the amount of chlorine in the tank after time

$Q'(t) = 0.10(200\ \%/\text{min}) - \frac{Q(200\ \%/\text{min})}{20{,}000}$

$Q'(t) = \ldots - Q(0.01)$

$0 = 20 - 0.01Q \qquad Q = 2000$ gallons

$Q_0(1200)$

$Q'(t) = 0.01(2000 - Q)$

$(\ln(2000 - Q))' = 0.01 \qquad 2000 - Q = Ce^{0.01t} \qquad Q(t) = 2000 - Ce^{0.01t}$

$1000 = 2000 - C$

$-1000 = C$

$Q'(t) = 2000 - 1000e^{0.01t}$

$\frac{2000}{1000} = e^{0.01t}$

A bowling ball with a drag coefficient of 0.8 is dropped from a tall building. Find $\frac{dv}{dt}$ $g = 9.8$ m/s²

$F_d = kv$

$F = F_g - F_d \qquad 5kg(9.8\ m/s^2) - (0.8)\,dv/dt$

$m\left(\frac{dv}{dt}\right) = 49 - 0.8v \qquad v' = 9.8\ m/s \quad 0.8\ dv/dt$

$\ln(61.25 - v)$

5/14/1?

$3y' + (T+4)y = T^2 + y'' \qquad$ linear $\qquad y''' = \cos(\pi T y) \qquad$ Not linear

$3y' + Ty + 4y = T^2 + y''$

when y is embedded in the function then its Not linear

linear or Non linear

ordinary or Partial

$f(x,y) = 3xy + 2x^2y$

$\frac{df}{dx} = 3y + 4xy$

$\frac{df}{dy} = 3x + 2x^2$

will get to these:
- Seperable
- exact
- Autonomous
- Homogeneous

Things Non-linear $(\text{Trig}(y),\ e^y,\ yy')$

$\underbrace{(x^2+3)y' + (2x)y}_{\uparrow} = x \qquad y' + P(x)y = g(x)$

$\int((x^2+3)y)' = \int x \qquad (x^2+3)y = \frac{x^2}{2} + C \qquad \boxed{y = \frac{x^2}{2x^2+6} + \frac{C}{x^2+3}}$

$e^{3x} \quad y' + 3y = x$

$y'e^{3x} + 3e^{3x}y = xe^{3x}$

$\int(e^{3x}y)' = \int xe^{3x}$

$e^{3x}y =$

integration by Parts $\int u\,dv = uv - \int v\,du$

$u = x \qquad du = dx$

$dv = e^{3x} \qquad v = \frac{e^{3x}}{3}$

$e^{3x}y = \frac{xe^{3x}}{3} - \int \frac{e^{3x}}{3}\,dx$

$\frac{xe^{3x}}{3} - \frac{e^{3x}}{9} + C$

$y = \frac{x}{3} - \frac{1}{9} + \frac{C}{e^{3x}}$