AAE 340 – Dynamics and Vibrations
Problem Set 11
Due: 11/20/13

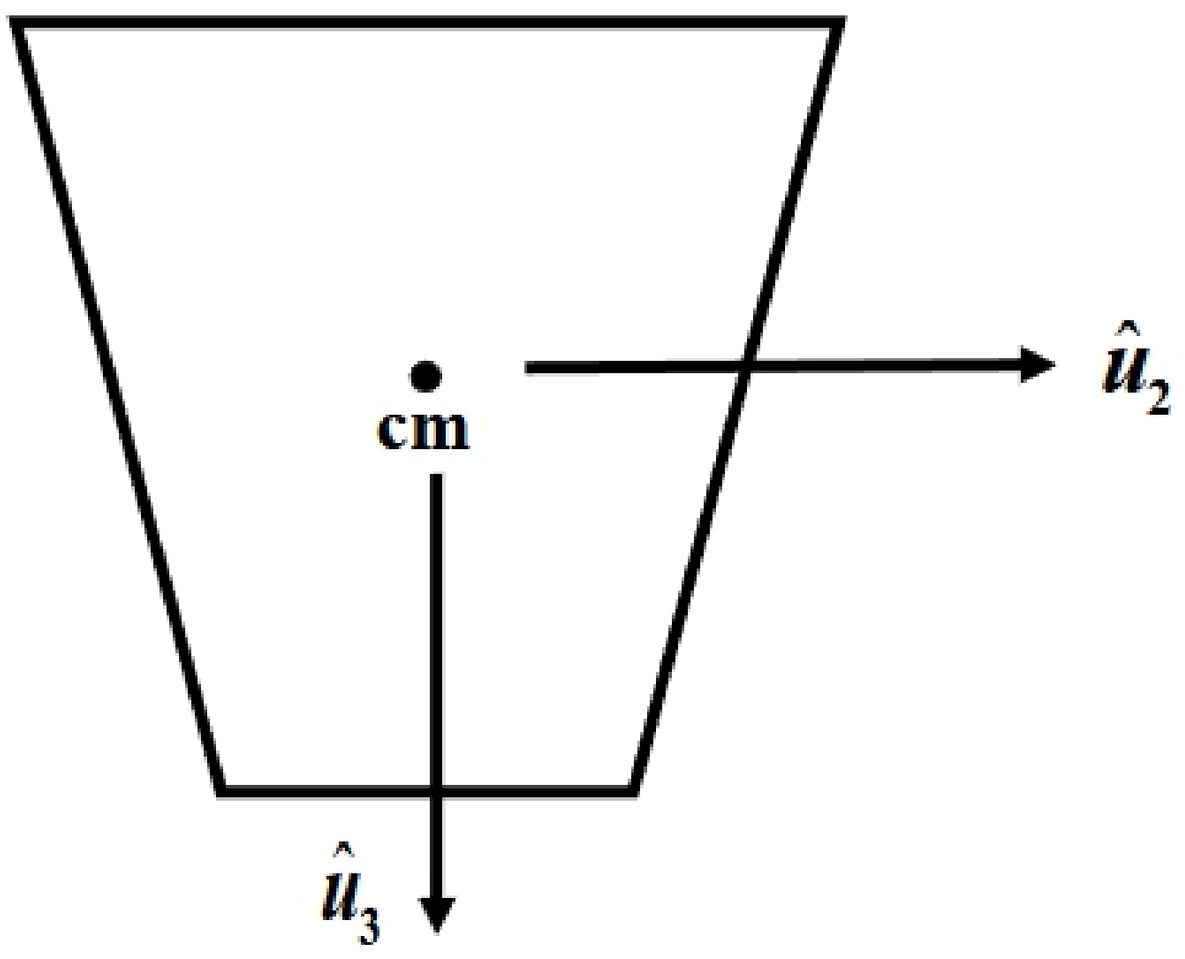
Problem 1: Below is a flat plate of non-uniform density. Inertia characteristics associated with the center of mass for body-fixed unit vectors are given

$$\left[I^{cm} \right]_{\hat{u}} = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 5 & -\sqrt{3} \\ 0 & -\sqrt{3} & 7 \end{bmatrix} \text{kg-m}^2$$

(a) Let $\hat{e}$ be unit vectors parallel to principal directions for the cm.

Determine $\left[I^{cm} \right]_{\hat{e}}$ and $\left[\ell \right]_{\hat{u} \cdot \hat{e}}$.

(b) Sketch the unit vectors $\hat{e}$ on the plate. Determine the angle between the $\hat{e}$'s in the plane of the plate and $\hat{u}_2, \hat{u}_3$.



Problem 2: Shown below is a thin, flat plate of uniform density $\nu = 0.75$ kg / met^2. Inertia characteristics associated with point Q for body-fixed vector basis $\hat{b}$ are given as

$$\left[I^{Q}_{\hat{b}}\right] = \begin{bmatrix} 128 & 0 & -48 \\ 0 & 208 & 0 \\ -48 & 0 & 80 \end{bmatrix} \text{ kg-m}^2$$

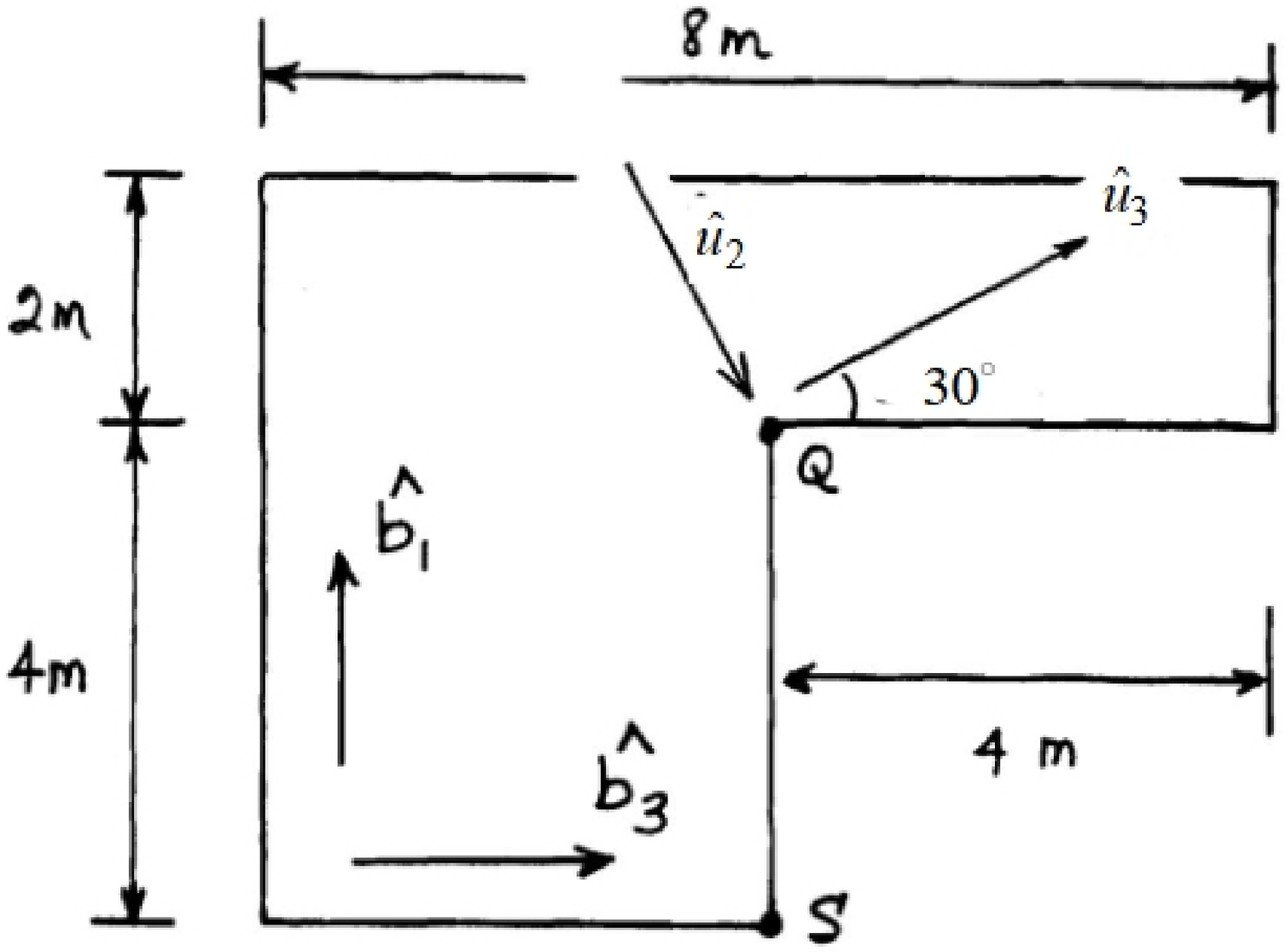
(a) Determine the direction cosine matrix $\left[\ell\right]_{\hat{u}\bullet\hat{b}}$. Locate the plate center of mass.

(b) Determine the elements of the inertia matrix for point S and associated with frame $\hat{u}$.

(c) Determine the principal moments and principal direction ($\hat{e}$) for point Q.

Write the direction cosine matrices $\left[\ell\right]_{\hat{e}\bullet\hat{b}}$ and $\left[\ell\right]_{\hat{u}\bullet\hat{e}}$.

Write the inertia matrix $\left[I^{Q}_{\hat{e}}\right]$. Verify the similarity transformation.



Problem 3: A uniform, thin rod of length L is welded to the bracket at point S on the underside of the (massless) disk D such that θ is a fixed (constant) angle. The disk rotates about a vertical axis with constant angular velocity ω.

(a) Determine the number of degrees of freedom for the motion of the rod. (Justify your answer.)

(b) Determine expressions for any moments that must be supported by the weld at point S.

$$\left\{ Ans: \ \Gamma_2 = -mg\frac{L}{2}\cos\theta + \frac{mL^2}{3}\omega^2 \sin\theta\cos\theta - \frac{mL^2}{8}\omega^2\sin\theta \right\}$$

(c) If $\theta = 76^\circ$, determine the values of ω that results in a zero moment at S.