Math 128 **Final Exam** **Spring 2009**

Name:
ID:
Discussion Section:

This exam consists of 16 questions:

- 14 multiple choice questions worth 5 points each
- 2 hand-graded questions worth a total of 30 points.

INSTRUCTIONS: Read each problem carefully and answer the question as written. You may use a non-graphing calculator and a standard sized (no larger than 4×6) index card worth of notes for the exam, but you may use no other aids. Record your answer to the multiple choice questions on the accompanying answer card. Show your work on the written problems and write clearly.

1. The Acme Widget Company has a marginal cost function given by $MC=50-3\sqrt{x+1}$. Suppose the total costs of production are \$8,000 per month at a production level of 99 widgets/month. What are the monthly **fixed** costs of production? (Choose the closest answer.)

(a) \$1,500.00

(b) \$2,050.00

(c) \$2,475.00

(d) \$2,950.00

(e) \$3,125.00

(f) \$3,333.00

(g) \$4,250.00

(h) \$5,050.00

(i) \$7,500.00

(j) \$8,000.00

Solution: (h). The total **variable** costs of production are given by:

$$\int_0^{99}(50-3\sqrt{x+1})\,dx=\left(50x-2(x+1)^{3/2}\right)\Big|_0^{99}=4950-2000=2950$$

So, the total **fixed** costs of production are \$8,000 - \$2,950 = \$5,050.

2. Find $\int_0^{\pi/2} x\sin x\,dx$.

(a) 0

(b) $1-\frac{\pi}{2}$

(c) $\frac{1}{2}$

(d) $\frac{1}{\sqrt{2}}$

(e) $\frac{\pi}{2}-1$

(f) 1

(g) $\sqrt{2}$

(h) $\frac{\pi}{2}$

(i) 2

(j) $\frac{\pi}{2}+1$

Solution: (f). Use Integration by Parts: $\int_0^{\pi/2} x\sin x\,dx=-x\cos x\Big|_0^{\pi/2}+\int_0^{\pi/2}\cos x=0+\sin x\Big|_0^{\pi/2}=1.$

3. Solve the IVP $(t^2+1)y'=t, y(0)=1$.

(a) $y=1$

(b) $y=\dfrac{1}{\sqrt{t^2+1}}$

(c) $y=t^2+1$

(d) $y=\dfrac{1}{2}t^2+1$

(e) $y=1+\ln(t^2+1)$

(f) $y=1+\ln(\sqrt{t^2+1})$

(g) $y=1-t$

Solution: (f). We see that $y'=\dfrac{t}{t^2+1}$. Integrating both sides yields $y=\int \dfrac{t}{t^2+1}dx$. Using the u-substitution $u=t^2+1, du=2t\,dt$ then gives

$$y=\frac{1}{2}\int \frac{1}{u}du=\frac{1}{2}\ln(|u|)+C=\frac{1}{2}\ln(t^2+1)+C=\ln(\sqrt{t^2+1})+C.$$

Now we solve for C:

$$1=y(0)=\ln(\sqrt{0+1})+C=0+C.$$

So, C = 1.