Dirk Bergemann and Larry Samuelson
Department of Economics
Yale University

Final Exam
Economics 501b
Microeconomic Theory

May 2008

This is a **closed-book** exam. The exam lasts for **180** minutes. Please write clearly and legibly. Be especially careful in the definition of the game, the payoff function and the equilibrium notions. The allocated points are also a good indicator for your time budget. Please record the answer for each question in a separate bluebook.

1. **(40)** Consider two firms (players 1 and 2) who are working on a joint project and a bank (player 3) who is a potential investor in the project. First, the entrepreneurs simultaneously decide whether to devote high or low effort to research on the project. They then make a presentation to the bank. If both firms choose high effort in their preliminary research, then the presentation goes well, and otherwise it goes poorly. The bank only observes the outcome of the presentation and not the firms' effort levels. After the presentation, the bank decides whether to invest in the project. Each firm receives a payoff of 5 if the bank invests and 0 otherwise. In addition, choosing high effort costs a firm 1, while choosing low effort is free. Investing costs the bank 2 and brings a return of 3 for each firm who chose high effort (i.e., a return of 6 if both chose high effort, 3 if only one did, and 0 if neither did). If the bank does not invest, his payoff is \$0. All players are risk neutral.

   (a) Draw an extensive form representation of this game.

   (b) Find all perfect Bayesian equilibria in which all players choose pure strategies.

   (c) Find all sequential equilibria in which all players choose pure strategies. Be precise about the difference between a perfect Bayesian and sequential equilibrium.

2. **(20)** Suppose that two players repeatedly play the following normal form game:

	T	M	B
T	$3,3$	$2,1$	$-2,4$
M	$1,2$	$1,1$	$-1,3$
B	$4,-2$	$3,-1$	$0,0$

   Suppose this game is infinitely repeated with a common discount factor $\delta \in (0,1)$, and that each player plays the following strategy:
   Begin play in stage I. Stage I: If T has always been played, play T. Otherwise, begin stage II. Stage II: If M has always been played since stage II began, play M; otherwise, begin Stage III. Stage III: Play B.

   (a) Present the automaton representation of these strategies. What is the payoff from playing these strategies?

   (b) For what values of the discount factor δ is this strategy profile a subgame-perfect equilibrium? What is the highest discount factor for which this outcome can be supported as the outcome of a subgame-perfect equilibrium? Substantiate your answer carefully, and be precise.

3. (**30**) Suppose an entrepreneur owns an assets which can take n different values, $a_i \in \{a_1, a_2, ..., a_n\}$ where, $a_1 < a_2 < \cdots < a_n$. The entrepreneur knows the value of the asset. He would like to sell the asset but outside investors only have a uniform probability distribution over all possible values a_i .The outside investors are always paying the expected value of the asset given their information. The agent can choose to *truthfully disclose* the value of the asset. (To be sure, he can either disclose the true value or not disclosing anything; in particular if he chooses to disclose, he cannot misrepresent the value of the asset.)
The game proceeds in three stages. First the entrepreneur chooses a disclosure policy, second the entrepreneur ask a price for the asset, third the outside investor accepts if the price is equal or below the expected value of the asset.

   (a) Define a strategy for entrepreneur and investor in this game. Define a Perfect Bayesian Equilibrium of this game.

   (b) Suppose there is no cost to disclosure. Derive a perfect Bayesian equilibrium of this game.

   (c) Suppose now that there is a positive cost of disclosure, say $K > 0$. Suppose further that $a_k - a_{k-1} > a_{k-1} - a_{k-2}$ for all k. How does the perfect Bayesian equilibrium strategy for the entrepreneur change with an increase in K. Present your arguments clearly.