EE468G NOTES (8B)

Reading assignment: Chapter 11

Contents: Transmission line: time harmonic analysis

Time harmonic wave on transmission line

The transmission line equation in frequency domain can be obtained by replacing $\partial/\partial t$ with $j\omega$:

$$\frac{\partial V}{\partial z} = -RI - j\omega LI = -(R + j\omega L)I$$

$$\frac{\partial I}{\partial z} = -GV - j\omega CV = -(G + j\omega C)I$$

In the above, V and I are complex functions of "z".

Lossless case: $$\begin{cases} \dfrac{\partial V}{\partial z} = -j\omega L I, & (1) \\ \dfrac{\partial I}{\partial z} = -j\omega C V, & (2) \end{cases}$$

Take $\partial/\partial z$ on both sides of eq. (1),

$$\frac{\partial^2 V}{\partial z^2} = -j\omega L \frac{\partial I}{\partial z} \quad \Longleftarrow\!=\!=\!= \text{use eq.(2) to replace } \frac{\partial \mathrm{I}}{\partial \mathrm{z}}$$

$$= -j\omega L\left(-j\omega CV\right) = -\omega^2 LCV$$

Let $\beta^2 = \omega^2 LC$, $\beta = \omega\sqrt{LC}$ [1/m] is the wavenumber. The equation for the voltage function V(z):

$$\frac{\partial^2 V}{\partial z^2} + \beta^2 V = 0$$

General solution:

$$V\left(z\right) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

Using eq.(1), we get the solution for the current I(z):

$$I\left(z\right) = \frac{1}{-j\omega L}\frac{\partial V}{\partial z} = \frac{-j\beta}{-j\omega L}\left[V^+ e^{-j\beta z} - V^- e^{j\beta z}\right]$$

$$= \frac{1}{Z_0}\left[V^+ e^{-j\beta z} - V^- e^{j\beta z}\right], \qquad Z_0 = \frac{\omega L}{\beta}$$

Parameters of a *lossless* transmission line:

Characteristic impedance: $Z_0 = \frac{\omega L}{\beta} = \frac{\omega L}{\omega\sqrt{LC}} = \sqrt{\frac{L}{C}}$, [Ω]

Wavenumber: $\beta = \frac{2\pi}{\lambda} = \omega\sqrt{LC}$ [rad/m] or [1/m]

Phase velocity: $u_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$

Group velocity: $u_g = \frac{d\omega}{d\beta}$

Special case for lossless TEM line: $LC = \mu\varepsilon$

For lossy line: the general solution can be written as:

$$V(z) = V^+ e^{-\alpha z} e^{-j\beta z} + V^- e^{\alpha z} e^{j\beta z}$$

α is the attenuation constant, its unit is [1/m] or [Np/m] (Np = Nepper). Sometimes, the attenuation constant is expressed in the unit of [dB/m]. The relation of the two units are:

$$\alpha \text{ [dB/m]} = 8.686 \times \alpha \text{ [Np/m]}$$

$$\alpha \text{ [Np/m]} = 0.1151 \text{ [dB/m]}$$

The attenuation constant is important for long-distance transmission. For example, if a cable has an attenuation constant of 0.3 [dB/km], then the attenuation would be 30 [dB] at 100 km distance away. This means the signal power is reduced by a factor of 1000.