5/12
page 1

Part 1: Modeling of Dynamic systems. - now to midterm

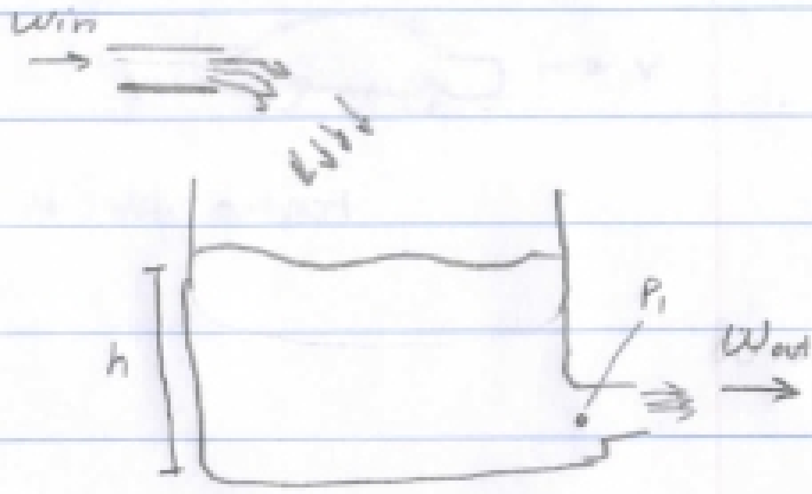
- Model 1st order systems by derivating their differential equations
- Model 2nd order systems by deriving their differential equations
- Model these systems using Transfer Functions
- Model these systems using State Space
- Use a model to predict response (transient & steady state)

What is a dynamic system?

- System
    - inputs
    - outputs
- Dynamic System: there is a delay between input & output
- Mathematical models of dynamic Systems
    - Assumptions
    - Limitations
- Validity of the model depends on validity of its assumptions
- Differential Equations: main way of modeling dynamic systems
- Feedback control
    - sensing + computation + actuation

1^{st} order dynamic system

- 1^{st} order: system has 1 state variable
- 1^{st} order: differential equation of system is order 1
- Example 1: a fluid tank



h_ know $h(t) = ?$

- Win
- Wout
- A
- Open/closed system
  ↳ pressure
- $h(0)$
- V^c

<u>Conservation of mass</u> → could be considered conservation of volume

$$\frac{dV}{dt} = W_{in} - W_{out}$$

$V = A \cdot h$ $\qquad W_{out} = c\,h$ ↗ a constant defining or face

$$A\frac{dh}{dt} = W_{in} - W_{out} \rightarrow \underline{A\dot{h} = W_{in} - ch} \Big\} \rightarrow 1^{st} \text{ order differential equation or } 1^{st} \text{ order dynamic system}$$

$$A\dot{h} + ch = W_{in}$$

$h(t) = x$ → state variable. 1 state variable thus 1^{st} order

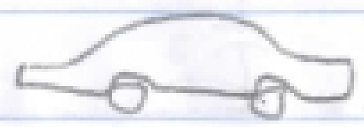
$W_{in} = U$ → input to the system

$h(t) = y$ → output of the system → need to know

Example 3 of a 1^{st} order system

input = force of engine

output = velocity, $V(t)$



need model to relate the input to the output

↳ write differential equation

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incline θ

m, F → input

Assumptions:

$-\theta = 0$, $F_f = 0$, $F_d = CV$

$$\frac{d}{dt} mV = \Sigma F \rightarrow m\frac{dV}{dt} = \Sigma F \rightarrow ma = \Sigma F$$

$$m\frac{dV}{dt} = F - CV \rightarrow \boxed{m\dot{V} + CV = F}$$

state variable = Velocity

Example 4 - Heat Transfer Example

- Assume an insulated space with heater
- $x(t) = T(t)$: State Variable
- $y = T$ → output variable
- $U = Q_{in}$ → input variable

need to know

$T(0)$, h, A, K, b, C_p

$$mC_p\frac{dT}{dt} = Q_{in} - Q_{out}$$

Assume:

only convection, $T_\infty = 0$

$$mC_p\dot{T} = Q_{in} - hA(T - \overset{0}{\cancel{T_\infty}}) \rightarrow \boxed{mC_p\dot{T} + hAT = Q_{in}}$$