Entanglement and Bell's Inequalities

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Abstract

Our goal for this experiment was to demonstrate violation of Bell's inequalties. This was accomplished by producing two polarization entangled photons via spontaneous parametric down-conversion using two type 1 beta barium borate crystals. With polarizers placed in front of two avalanche photodiodes, we observed how the alignment of the polarizers affected the number of observed photon pairs coincident on the photodiodes. The data obtained in this way was used to construct the S value described by the Clauser-Horne-Shimony-Holt inequality. This inequality states that S is no greater than 2, but we observed a value of $S = 2.64$. Thus, we demonstrated violation of a Bell's inequality, refuting the family of local realist theories of quantum mechanics.

1 Background

In quantum theory, two particles are considered entangled if their state is not separable: $|\Psi_{12}\rangle \neq |\Psi_1\rangle \otimes |\Psi_2\rangle$. The state of an entangled particle cannot be specified without reference to the other particle. By the Copenhagen interpretation of quantum mechanics, when the state of one entangled particle is measured, the wave function collapses and the state of the other particle is also determined. Such a collapse is supposedly instantaneous, and this idea of a non-local effect historically led to objections from detractors who preferred that quantum mechanics be described by a more classical local hidden variable theory.

To illustrate the difference in these view points, consider the case of Bertlmann and his socks, as posed by Bell. Bertlmann always wears different colored socks. If you observe that he has a pink sock on one foot, you know the other sock he is wearing is not pink, and this information is gained instantly without needing to observe the other sock.

Contrast Bertlmann's socks with the case of entangled particles. In quantum mechanics, entangled particles are treated as being in a superposition of possible states. We cannot be certain of what state we will measure a particle as being in, and by the Copenhagen interpretation, we cannot even speak meaningfully of a definite state measurement result because the state of each particle is undetermined prior to observation. Upon observation the state of both particles collapse instantly to a single state. This differs from the case of Bertlmann in two crucial ways: 1) Bertlmann was wearing a pink sock regardless of whether you observed his socks or not and 2) the observation of Bertlmann wearing one pink sock did not affect the color of the other sock[1]. In particular, the concept that an observation of one particle state could instantly alter the state of another particle regardless of spatial separation provides the basis for the objection to quantum mechanical formalism given by Einstein, Poldolsky, and Rosen[2], who disputed the existence of such a "spooky action at a distance"[3].

Bell's inequalities are significant in that they allow experimental testing of the opposing quantum and classical interpretations. A Bell's inequality is simply a mathematical inequality derived by assuming locality and conterfactual definiteness, conterfactual definiteness referring to the realist view that the state of particles are determined by some hidden variables that give a measurement of the state a definite result. Bell's inequalities can be calculated for classical quantities, but inequalities for classical examples are obviously upheld and therefore not of great interest. Such inequalities suddenly become interesting when considering quantum cases, where it is predicted and observed that these inequalities are violated[4]. Observed violation of Bell's inequalities leads to the conclusion that a local realist theory cannot be used to reproduce all of the predictions of quantum mechanics, as stated by Bell's theorem[5].

2 Theory

The Bell's inequality relevant to this experiment is the Clauser-Horne-Shimony-Holt(CHSH) inequality[6]. First, consider two polarization entangled particles, in the state $|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}(|V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i)$, where V and H refer to vertical and horizontal configurations and s and i are the historical way to label photons produced by spontaneous parametric down-conversion. This state is invariant regardless of choice of polarization basis. To show this, consider a rotation by an angle α, so we will make use of the rotated basis states of $|V_\alpha\rangle = \cos(\alpha)|V\rangle + \sin(\alpha)|H\rangle$ and $|H_\alpha\rangle = -\sin(\alpha)|V\rangle + \cos(\alpha)|H\rangle$. In this basis,

$$\begin{aligned}
\frac{1}{\sqrt{2}}(|V_\alpha\rangle_s|V_\alpha\rangle_i + |H_\alpha\rangle_s|H_\alpha\rangle_i) &= \frac{1}{\sqrt{2}}[(\cos(\alpha)|V\rangle_s + \sin(\alpha)|H\rangle_s)(\cos(\alpha)|V\rangle_i + \sin(\alpha)|H\rangle_i) \\
&+ (-\sin(\alpha)|V\rangle_s + \cos(\alpha)|H\rangle_s)(-\sin(\alpha)|V\rangle_i + \cos(\alpha)|H\rangle_i)] \\
&= \frac{1}{\sqrt{2}}[\cos^2(\alpha)|V\rangle_s|V\rangle_i + \sin^2(\alpha)|H\rangle_s|H\rangle_i \\
&+ \sin(\alpha)\cos(\alpha)|V\rangle_s|H\rangle_i + \sin(\alpha)\cos(\alpha)|H\rangle_s|V\rangle_i \\
&+ \sin^2(\alpha)|V\rangle_s|V\rangle_i + \cos^2(\alpha)|H\rangle_s|H\rangle_i \\
&- \sin(\alpha)\cos(\alpha)|V\rangle_s|H\rangle_i - \sin(\alpha)\cos(\alpha)|H\rangle_s|V\rangle_i] \\
&= \frac{1}{\sqrt{2}}(|V\rangle_s|V\rangle_i + |H\rangle_s|H\rangle_i) \\
&= |\Psi_{Bell}\rangle
\end{aligned}$$

so $|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}(|V_\alpha\rangle_s|V_\alpha\rangle_i + |H_\alpha\rangle_s|H_\alpha\rangle_i)$. Thus, $|\Psi_{Bell}\rangle$ is invariant under rotation.

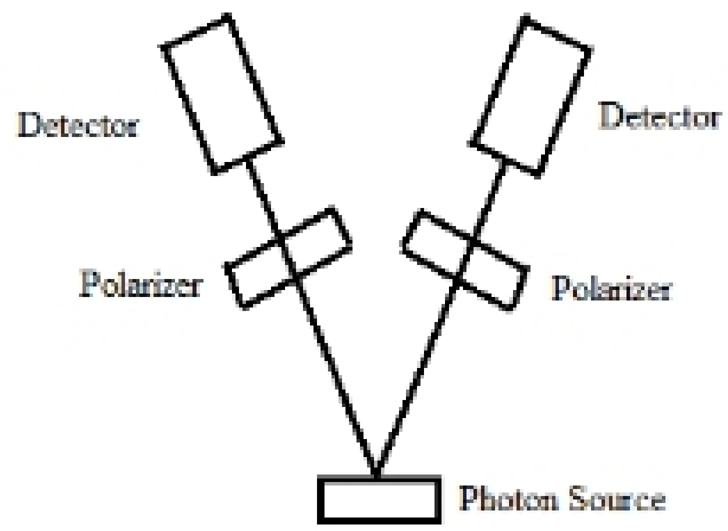

Figure 1: Outline of the thought experiment. Polarization entangled photon pairs are emitted from a source, pass through polarizers and are received by a detector.

Consider now measuring the polarization of these photons. As outlined in Figure 1, imagine situating two detectors so that each detector will receive one of the entangled photons, and then imagine placing a polarizer in front of each detector, one polarizer rotated to the arbitrary angle α, the other to the angle β. The probability that both photons are vertical in the bases of their respective polarizers is

$$
\begin{aligned}
P_{VV}(\alpha,\beta) &= |\langle V_\alpha|_i \langle V_\beta|_s |\Psi_{Bell}\rangle|^2 \\
&= \frac{1}{2}|[\langle V|_i \cos(\alpha) + \langle H|_i \sin(\alpha)][\langle V|_s \cos(\beta) + \langle H|_s \sin(\beta)][|V\rangle_i |V\rangle_s + |H\rangle_i |H\rangle_s]|^2 \\
&= \frac{1}{2}|\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)|^2 \\
&= \frac{1}{2}\cos^2(\alpha-\beta).
\end{aligned}
$$

Similarly, the probabilities for other polarization measurements are

$$
\begin{aligned}
P_{HH}(\alpha,\beta) &= \frac{1}{2}\cos^2(\alpha-\beta) \\
P_{VH}(\alpha,\beta) &= \frac{1}{2}\sin^2(\alpha-\beta) \\
P_{HV}(\alpha,\beta) &= \frac{1}{2}\sin^2(\alpha-\beta).
\end{aligned}
$$

Defining $N(\alpha,\beta)$ as the number of coincident photon counts when the polarizers are aligned at angles α and β, it is obvious that

$$
\begin{aligned}
P_{VV}(\alpha,\beta) &= \frac{N(\alpha,\beta)}{N_{Total}} \\
P_{HH}(\alpha,\beta) &= \frac{N(\alpha_\perp,\beta_\perp)}{N_{Total}} \\
P_{VH}(\alpha,\beta) &= \frac{N(\alpha,\beta_\perp)}{N_{Total}} \\
P_{HV}(\alpha,\beta) &= \frac{N(\alpha_\perp,\beta)}{N_{Total}},
\end{aligned}
$$

where $\alpha_\perp = \alpha + 90°$ and $\beta_\perp = \beta + 90°$. N_{Total} refers to the total number of coincident pairs detected: $N_{Total} = N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta)$. Knowing this, it is easy to measure the probabilities P_{VV}, P_{HH}, etc. by simply measuring the number of coincident photon pairs detected when the polarizers are aligned appropriately.

Consider now the correlation function

$$
\begin{aligned}
E(\alpha,\beta) &= P_{VV} + P_{HH} - P_{VH} - P_{HV} \\
&= \frac{1}{2}\cos^2(\alpha-\beta) + \frac{1}{2}\cos^2(\alpha-\beta) - \frac{1}{2}\sin^2(\alpha-\beta) - \frac{1}{2}\sin^2(\alpha-\beta) \\
&= \cos^2(\alpha-\beta) - \sin^2(\alpha-\beta) \\
&= \cos(2(\alpha-\beta)).
\end{aligned}
$$

With this quantity in mind, we can make use of the Clauser-Horne-Shimony-Holt inequality.

The Clauser-Horne-Shimony-Holt (CHSH) inequality concerns the quantity S defined as

$$
S = |E(a,b) - E(a,b')| + |E(a',b) + E(a',b')|.
$$