```
> restart;
```

Differential Equations

In this worksheet we demonstrate basic techniques of working with *ordinary differential equations*, (ODE's), using Maple. An ODE is a differential equation in which the unknown function is a function of one variable. We shall stick with *first order* ODE's which contain only the first derivative of the unknown function. Moreover, we shall consider only first order ODE's *in the normal form*, that is in the form:

$$\frac{y}{x} = f(x, y)$$

,

where the left hand side is simply the derivative of the unknown function, $y = y(x)$ in this case, and the right hand side is a given function of x and y. You know that for such ODE's, under reasonable assumptions about $f(x, y)$, there exists a unique solution to any *initial value problem*, (IVP), of the form

$$\frac{y}{x} = f(x, y)$$

$, y(x_0) = y_0$,

where x_0 , y_0 are given.

0.1 Solving and Plotting ODE's

Basic tools for solving and plotting ODE's are contained in the packages "plots" and "DEtools". We begin with loading these packages. Please, don't forget to click on the two commands below.

```
> with(plots):
> with(DEtools):
```

We are familiar with the package "plots". If you are curious about the content of "DEtools", replace the colon at the end of the command with a semicolon and click on it again.

Example 1.(a) Find the general solution to the ODE: $\frac{y}{x} = -2\,xy$.

(b) Solve the following two initial value problems:

$$\frac{y}{x} = -2\,xy$$

$, y(0) = 2$,

and

$$\frac{y}{x} = -2\,xy$$

$, y(0) = 1/2$.

(c) Plot the solutions to the IVP's together with the slope field corresponding to the ODE.

In order to simplify many commands below let's first label our ODE:

```
> ODE1:=diff(y(x),x)=-2*x*y(x);
```

$$ODE1 := \frac{d}{dx}y(x) = -2\,xy(x)$$

Observe that correct syntax. The derivative is entered using the "diff" command. The command "D(y)(x)" could be used as well. Note that "y" has to be entered as "y(x)". The main command for solving ODE's is "dsolve".

```
> dsolve(ODE1,y(x));
```

$$y(x) = \_C1\,e^{-x^2}$$

Maple returned the general solution. "_C1" denotes, of course, an arbitrary constant. Instead of using the name "ODE1" you could have entered the differential equation "diff(y(x),x)=-2*x*y(x)" directly into the "dsolve" command. Maple can handle initial value problems, as well. The proper syntax looks as follows.

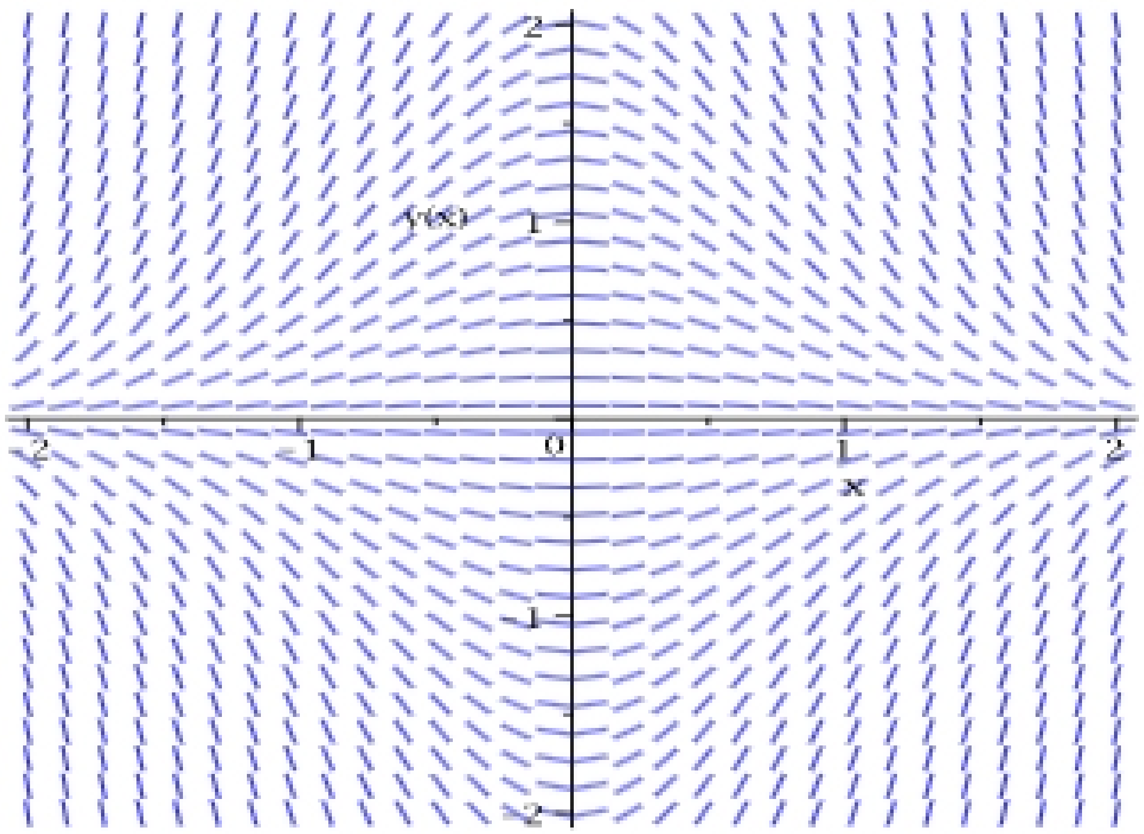
```
> dsolve({ODE1,y(0)=2},y(x)); dsolve({ODE1,y(0)=1/2},y(x));
```

$$y(x) = 2\,e^{-x^2}$$

$$y(x) = 1/2\,e^{-x^2}$$

Maple can plot slope fields, as well as slope fields together with particular solutions. Proper commands are "dfieldplot" and"DEplot", both contained in the "DEtools" package. Let's see how they work. Pay attention to the syntax.

```
> dfieldplot(ODE1,[y(x)],x=-2..2,y=-2..2,color=blue,scaling=constrained,arrows=LINE,dirgr
```



Maple plotted the slope field for our equation. All the options under the "dfieldplot" command regarding color, appearance of the arrows, scaling and dirgrid are, of course, optional. You can play with them and see what will happen. "dirgrid" tells Maple how dense you want the field of slopes to be. The default setting is dirgrid[20,20] and it tends to be a little rough. On the other hand, a finer grid may take more time to compute.

Remark. Whenever plotting field of slopes, you should use the "scaling = constrained" option. Otherwise, the pictures may appear misleading, as slopes become distorted.

The command "DEplot" plots the slope field together with particular solutions.

```
> DEplot(ODE1,y(x),-2..2,[[y(0)=2],[y(0)=1/2]], linecolor=magenta,color=blue,scaling=cons
```