9/18/14

vector notation $\langle 1,2,3\rangle = \begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$

$[1\ 2\ 3] = (1\ 2\ 3)$ These not same as

Difference is no commas.

$$[1\quad 2\quad 3]\begin{bmatrix}4\\5\\6\end{bmatrix} = (1)(4)+(2)(5)+(3)(6) = 32$$

$$\begin{bmatrix}4\\5\\6\end{bmatrix}[1\ 2\ 3] \neq [1\ 2\ 3]\begin{bmatrix}4\\5\\6\end{bmatrix}$$

$$r(t) = \begin{bmatrix}x(t)\\y(t)\\z(t)\end{bmatrix} = \begin{bmatrix}t^3\\3t^2+t\\t^5+8\end{bmatrix}$$

$f(x,y,z) = x^4y^2 + y^4z + xz^3$

The derivative of f

$p = (x,y,z)$

$\Delta p = (\Delta x, \Delta y, \Delta z)$

$$f(p+\Delta p) = f(p) + f'(p)\Delta p + \epsilon(p,\Delta p)$$

* when you plug in point

$$[f_x(p)\ f_y(p)\ f_z(p)] \quad \begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix}$$

$$\left[\tfrac{\partial f}{\partial x}\quad \tfrac{\partial f}{\partial y}\quad \tfrac{\partial f}{\partial z}\right]$$

$$\approx f(p) + f_x(p)\Delta x + f_y(p)\Delta y + f_z(p)\Delta z$$

$f_x(x,y,z) = 4x^3y^2 + z^3$

$f_y(x,y,z) = 2x^4y + 4y^3z$

$f_z(x,y,z) = y^4 + 3xz^2$

$$f'(2,1,3) = \cancel{\begin{bmatrix} 4(2)^3(1)^2 + (3)^3 \\ 2(2)^4(1) + 4(1)^3(3) \\ (1)^4 + 3(2)(3)^2 \end{bmatrix} = \begin{bmatrix} 59 \\ 44 \\ 55 \end{bmatrix}}$$

$$\begin{bmatrix} 59 & 44 & 55 \end{bmatrix}$$

$f: \mathbb{R}^3 \to \mathbb{R}$

$f(x,y,z)$ 3 input, 1 output

$\text{output}\{(\overbrace{\qquad\qquad}^{\text{input}})$

$f: \mathbb{R}^3 \to \mathbb{R}^1 \to [\qquad]$

$f: \mathbb{R}^1 \to \mathbb{R}^3 \to$

Clairut

H f, f_x, f_y

f_{xy}, f_{yx}

all exist & are continuous in a neighborhood of P

then $f_{xy}(P) = f_{yx}(P)$.

(Unless they're cooked up special & but we don't need to worry about them)

Chain Rule

$$(f\circ g)' = (f'\circ g)(g')$$

$$(f\circ g)'(P) = (f'\circ g)(P) \cdot g'(P)$$
$$= f'(g(P)) \cdot g'(P)$$

$$g(t) = \begin{bmatrix} t^3+t \\ 2t^2+8 \\ t^4 \end{bmatrix} \qquad \mathbb{R}^1 \xrightarrow{\ g\ } \mathbb{R}^3 \xrightarrow{\ f\ } \mathbb{R}^1$$

$(f\circ g)$

$$f'(x,y,z) = [f_x(x,y,z) \quad f_y(x,y,z) \quad f_z(x,y,z)]$$
$$= [4x^3y^2+z^3 \quad 2x^4y+4x^3z \quad y^4+3xz^2]$$

$$g'(t) = \begin{bmatrix} 3t^2+1 \\ 4t \\ 4t^3 \end{bmatrix}$$

$$(f\circ g)'(t) = f'(g(t))\, g'(t)$$

$$f'(g(t)) = [4(t^3+t)^3(2t^2+8)^2+(t^4)^3 \quad 2(t^3+t)^4(2t^2+8)+4(t^3+t)^3(t^4) \quad (2t^2)^4+3(t^3+t)(t^4)^2]$$

$$(f\circ g)'(t) = [4(t^3+t)^3(2t^2+8)^2+(t^4)^3 \quad 2(t^3+t)^4(2t^2+8)+4(t^3+t)^3(t^4) \quad (2t^2)^4+3(t^3+t)(t^4)^2] \cdot \begin{bmatrix} 3t^2+1 \\ 4t \\ 4t^3 \end{bmatrix}$$

$$(f\circ g)'(t) = (4(t^3+t)^3(2t^2+8)^2+(t^4)^3)(3t^2+1)$$
$$+ (2(t^3+t)^4(2t^2+8)+4(t^3+t)^3(t^4))(4t)$$
$$+ ((2t^2)^4+3(t^3+t)(t^4)^2)(4t^3)$$

$$(f\circ g)'(0) = 0$$

$(f\circ g)'(1) = (4\cdot 8\cdot 100+1)(4)+(2)(2^4)(10)$ … keep going…