Methods to Estimate Dynamic Stochastic General Equilibrium Models

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Abstract

This paper employs the one-sector Real Business Cycle model as a testing ground for four different procedures to estimate Dynamic Stochastic General Equilibrium (DSGE) models. The procedures are: 1) Maximum Likelihood, with and without measurement errors and incorporating Bayesian priors, 2) Generalized Method of Moments, 3) Simulated Method of Moments, and 4) Indirect Inference. It is shown that stochastic singularity limits the number of variables and moments that can be used for the estimation of DSGE models and, in general, affects more severely Maximum Likelihood than other estimation procedures. Monte Carlo analysis indicates that all methods deliver reasonably good estimates under the null, but that moment-based procedures are substantially more robust to misspecification than Maximum Likelihood.

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1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models have become a standard tool in various fields of Economics, most notably in Macroeconomics and International Economics. DSGE models are attractive because they explicitly specify the objectives and constraints faced by households and firms, and then determine the prices and allocations that result from their market interaction in an uncertain environment.

To date, calibration is by far the most common approach in the literature to examine the empirical properties of DSGE models. In calibration, the value of the structural parameters is fixed to those estimated in previous microeconometric studies and/or those computed using long-run averages of aggregate data. Then, the model is simulated using a synthetic series of shocks, and the unconditional moments of the simulated economic series are computed and compared with those of actual data. The model is usually evaluated in terms of the distance between these two set of moments. This comparison may be casual or based on measures of fit like the ones proposed, for example, by Gregory and Smith (1991), Watson (1993), and DeJong, Ingram, and Whiteman (1996). Impulse-response analysis and variance decomposition are also used to examine, respectively, the model's behavior following exogenous shocks and to assess the relative importance of these shocks in explaining the conditional and unconditional variances of the variables.

Although calibration is a very useful tool for understanding the dynamic properties of DSGE models, there are some advantages in their fully-fledged econometric estimation. First, parameter estimates are obtained by imposing on the data the restrictions of the model of interest. This addresses the concern that the assumptions of the DSGE model might be inconsistent with the assumptions employed by the micro studies that produced the parameter estimates used in calibration. Second, the estimation of the DSGE model allows one to obtain estimates of parameters that might be hard to estimate using disaggregated data alone. Third, parameter uncertainty may be explicitly incorporated in impulse-response analysis using, for example, bootstrap techniques to construct confidence intervals for the model's response to a shock. Finally, standard tools of model selection and evaluation can be readily applied. For example, one can test the residuals for serial correlation and neglected Autoregressive Conditional Heteroskedasticity, compare the Root Mean Square Error of the DSGE model with that of another DSGE model or a Vector Autoregression, perform tests of parameter stability or directly test some of the model's identification assumptions. All this is valuable information in the construction of more realistic economic models.[1]

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[1] See Hansen and Heckman (1996), and Browning, Hansen, and Heckman (1999) for additional discussion. For a defense of the merits of calibration, see Kydland and Prescott (1996).

The estimation procedures studied here are Maximum Likelihood (ML), Generalized Method of Moments (GMM), Simulated Method of Moments (SMM), and the Indirect Inference procedure proposed by Smith (1993). All these procedures are standard and their asymptotic properties are well known. The goals of this paper are to describe in a pedagogical manner their application to the estimation of DSGE models, to study their small-sample properties, to examine their robustness to misspecification, to compare their computational costs, and to discuss fully the implications of the stochastic singularity of DSGE models for each estimation procedure. The intention here is not to perform a "horse race" between different estimation strategies. Instead, the more constructive goal is to evaluate their relative strengths and weaknesses in the context of a simple, but economically interesting model.

A feature of linearized DSGE models that has important implications for all estimation methods is their stochastic singularity. Linearized DSGE models are singular because they generate predictions about more observable variables than exogenous shocks are specified in the model.[2] Thus, these models imply that certain linear combinations of observable variables should hold without noise. This implication is not satisfied by actual data and arises from a particular form of misspecification, namely that models assume a smaller number of shocks than are present in the real world.

Stochastic singularity limits the number of variables and moments that can be used for the estimation of the model, and imposes restrictions on both the order and the number of variables in the VAR representation of artificial data generated by a DSGE model. In general, singularity affects more severely Maximum Likelihood than the other estimation methods. ML estimation is limited by the number of linearly independent variables, while Method of Moments and Indirect Inference are limited by the number of linearly independent moments. That latter is a weaker restriction because it is possible to find independent moments that incorporate information about more variables than those that are linearly independent. For example, the RBC model studied here cannot be estimated by Maximum Likelihood using more than one variable, unless measurement errors are added, but it can be estimated by the Method of Moments or Indirect Inference using moments that involve two variables. This observation has two implications. First, the efficiency gain of using ML rather than Method of Moments or Indirect Inference may be modest. Second, identification may be sharper under either of the latter procedures than under ML because information on a larger set of variables can be employed.

Monte Carlo analysis is used to study the small-sample properties of the estimators

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[2] Although strictly-speaking nonlinear DSGE models are not singular, the discussion in Section 3.4 makes clear that their estimation may also be affected depending on the extent to which they differ from their linearized counterparts.