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Evaluation of approximate methods to estimate maximum inelastic displacement demands

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SUMMARY

Six approximate methods to estimate the maximum inelastic displacement demand of single-degree-of-freedom systems are evaluated. In all methods, the maximum displacement demand of inelastic systems is estimated from the maximum displacement demand of linear elastic systems. Of the methods evaluated herein, four are based on equivalent linearization in which the maximum deformation is estimated as the maximum deformation of a linear elastic system with lower lateral stiffness and with higher damping coefficient than those of the inelastic system. In the other two methods the maximum inelastic displacement is estimated as a product of the maximum deformation of a linear elastic system with the same lateral stiffness and the same damping coefficient as those of the inelastic system for which the maximum displacement is being estimated, times a modifying factor. Elastoplastic and stiffness-degrading models with periods between 0.05 and 3.0 s are considered when subjected to 264 ground motions recorded on firm sites in California. Mean ratios of approximate to exact maximum displacements corresponding to each method are computed as a function of the period of vibration and as a function of the displacement ductility ratio. Finally, comments on the advantages and disadvantages of each method when applied to practical situations are given. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: displacement-based design; performance-based design; approximate methods; inelastic displacement; elastic displacement; equivalent-linear systems; inelastic displacement ratios

1. INTRODUCTION

Practically all structural damage and a large portion of the non-structural damage sustained in buildings as a result of earthquake ground motions is produced by lateral displacements. Thus, the estimation of lateral displacement demands is of primary importance in performance-based earthquake resistant design and in general when damage control is of interest.

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Furthermore, most structures will experience inelastic deformations when subjected to severe earthquake ground motions. Thus, of special interest is an adequate estimation of lateral displacement demands in structures that exhibit non-linear behaviour. This is particularly true in performance-based design in which a better prediction of seismic performance is desired.

When the full characterization of the ground motion is available maximum displacement demands can be computed through time-history analysis. However, in most cases the design of new structures or the evaluation and upgrading of existing structures is not carried out with time history analyses, instead seismic demands are specified with the maximum response of linear elastic single-degree-of-freedom (SDOF) using design linear elastic response spectra or uniform-hazard linear elastic response spectra. Thus, approximate methods to estimate the maximum inelastic displacement demands from the maximum displacement demand of linear elastic SDOF systems are particularly useful in these situations. For example, several recently proposed displacement-based methods [1–8] use the response of linear elastic SDOF systems to estimate the maximum inelastic displacements in bridge and building structures. Similarly, recently published design recommendations [9–11] include analysis procedures where global lateral displacement demands on structures, usually referred to as target displacements, are computed from maximum deformations of linear elastic SDOF systems. In all of these approximate methods a key step is the estimation of maximum inelastic displacement demand of SDOF systems from the maximum displacement demand of linear elastic SDOF systems. Moreover, these approximate analysis methods provide useful insight to the response of inelastic systems to earthquake ground motions that is difficult to obtain from the response time history computed from an individual record. Hence, the evaluation of available approximate methods to estimate maximum inelastic displacement demands of SDOF systems from maximum displacement demands of elastic SDOF systems is specially valuable for users of these recently proposed displacement-based procedures.

The objective of this work is to evaluate six approximate methods to estimate maximum inelastic displacement demands of SDOF systems from maximum displacement demands of elastic SDOF systems which form the underlying principle of most of the approximate analysis procedures used in recently proposed displacement-based design methods. It is beyond the scope of this work to evaluate the use of these methods to estimate inelastic displacement demands of multi-degree-of-freedom (MDOF) systems and to evaluate the actual implementation of the approximate SDOF methods in displacement-based design procedures.

Although there have been several studies [12–15] that have evaluated approximate methods to estimate maximum inelastic displacement demands of SDOF systems, their scope has been very limited. For example Jennings [12] summarized and compared six early proposals of equivalent linearization methods but results were primarily based on harmonic loading and provided no conclusions regarding the accuracy of any of the methods. Hadjian [15] performed an evaluation of seven equivalent linear methods. Although his study provided interesting comparisons and comments regarding various methods when used under harmonic and earthquakes loads, the study did not provide any quantitative results regarding the accuracy of the results that can be expected from the user of these methods. Perhaps the bes evaluation conducted to date was done by Iwan [13, 14] who evaluated various approximate methods. However, the results only considered 12 earthquake ground motions, three levels of inelastic behaviour and were restricted to nine mid-range periods between 0.4 and 4.0 s, hence no evaluation for short period structures was provided. Error measures provided for

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each method were averaged through various periods, thus, information regarding errors for specific periods of vibration was not provided. Furthermore, the error measure used, although provided quantitative results of the size of the averaged errors corresponding to each method, did not provide information whether the approximate method tended to overestimate or underestimate the maximum displacements. Moreover, some of the approximate methods that are being implemented in recent displacement-based design recommendations were not available when these previous evaluation studies were made. Of particular interest to practicing engineers is to know which approximate methods, that provide the basis in recently proposed analysis procedures, produce better results for specific periods of vibration or at least for specific spectral regions as well to know which methods provide better results for specific levels of inelastic behaviour expected to occur in the structure. For example, some methods may provide better results in the short period region than others. Similarly some methods may provide better results for higher levels of inelastic behaviour than others. Equally important to practicing engineers is to be aware of the limitations of these approximate methods to estimate maximum inelastic displacements, and in particular to be aware of the level of errors that can be produced while using these approximate methods.

2. APPROXIMATE METHODS TO ESTIMATE MAXIMUM INELASTIC DISPLACEMENT DEMANDS

Many approximate methods to estimate maximum inelastic displacement demands from maximum elastic displacement in SDOF systems have been proposed. In general, approximate methods commonly used can be classified into two main groups. The first group comprises methods based on equivalent linearization in which the maximum deformation is estimated as the maximum deformation of an equivalent linear elastic system with lower lateral stiffness (higher period of vibration) and with higher damping coefficient than those of the system for which the maximum inelastic displacement is being estimated. The second group includes methods in which the maximum inelastic displacement is estimated as a product of the maximum deformation of a linear elastic system with the same lateral stiffness and same damping coefficient than that of the inelastic system for which the maximum displacement is being estimated times a displacement modification factor.

2.1. Methods based on equivalent linearization

The concept of equivalent viscous damping was first proposed by Jacobsen [16] to obtain approximate solutions of the steady forced vibration of damped SDOF systems with linear force–displacement relationships but with damping forces proportional to the nth power of the velocity of motion when subjected to sinusoidal forces. In this pioneering study, the stiffness of the equivalent system was set equal to the stiffness of the real system and the equivalent viscous damping ratio was based on equating the dissipated energy per cycle of the real damping force to that of the equivalent damping force. Years later, the same author extended the concept of equivalent viscous damping to yielding SDOF systems [17] by considering simultaneously an equivalent viscous damping ratio and a period shift. When a period shift is used many different values of equivalent viscous damping can be obtained depending on the selection of the period shift. As noted by Jennings [12] and Hadjian [15] if the equal energy

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