

**Section 7: Nonparametric uncertainty
estimation**

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Section 17.

27 Nov. 2001

Nonparametric methods of uncertainty estimation

δ -method, jackknife, bootstrap, cross-validation

Techniques broadly applicable, complex statistics

Generally justified by asymptotics

There exist singular and inappropriate cases

NECESSARY

δ -method AKA method of linearization, propagation of error, Taylor series method

Gauss (1815)

Basically one approximates functions by Taylor expansions (usually linear) of basic random variables.

Rao, Section 6a.2

Sequence of k -dimensional statistics

$$T_n = (T_{1n}, \dots, T_{kn}) \quad n = 1, 2, \dots$$

eg. (\bar{X}, \bar{Y})

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27 Nov 2001

Derived statistic

$$g(\underline{T}_n) = g(T_{1n}, \dots, T_{kn})$$

eg. $g(\bar{X}, \bar{Y}) = \frac{\bar{Y}}{\bar{X}}$

Suppose

$$\sqrt{n}(\underline{T}_n - \underline{\theta}) \xrightarrow{d} N_p(0, \underline{\Sigma}) \xrightarrow{L}$$

Write

$$g(\underline{T}_n) = g(\underline{\theta}) + \frac{\partial g(\underline{\theta})}{\partial \underline{\theta}} (\underline{T}_n - \underline{\theta}) + \dots$$

The entity of principal interest is typically $g(\underline{\theta})$

Theorem. If $g(\cdot)$ has a continuous first derivative

$$\sqrt{n} \{g(\underline{T}_n) - g(\underline{\theta})\} \xrightarrow{d} N_p\left(0, \frac{\partial g}{\partial \underline{\theta}} \underline{\Sigma} \frac{\partial g}{\partial \underline{\theta}}\right)$$