Economics 402 Exam 4, April 17 (buff version)

You have one hour and fifteen minutes for this exam. Please do not use anything other than writing implements (pen, pencil, eraser), i.e. no electronics, calculator, etcetera. Please answer the questions on this exam paper itself. Please do not ask the proctor any questions to ensure that everyone can write their exam in a quiet disruption–free environment. You may leave when you're done. The number of points for each question is indicated in square brackets following the question.

Please show enough of your work to convince the grader that you know what you're doing. There are 22 points available on the exam; 20 points=100%.

1. Please consider the following table.

Benefit to consumers		
	April	May
High:	8	2
Low:	3	1

There are two types of consumers of some consumer product A: high and low. There are two high value consumers and three low value consumers. The benefit in monetary terms that high and low value consumers derive from consuming product A in each month is as indicated in the table. Thus, high value consumers get \$2 worth of utility (each) if they purchase the product in May and \$10 (8+2) worth of utility if they purchase the product in April. The seller S only cares about total revenue. She can set one price for all consumers in April and one price for all consumers in May, but the prices that she sets in each month can be different. All information is known to everyone. Assume sequential rationality.

(a) How much would S charge in April? **[3]**

S will set a price of \$1 in May regardless of what happens in April. If she charges \$4 in April then she gets $5 \times 4 = 20$. If she charges more than \$9 in April then no one will buy since waiting yields a payoff of \$1 for high value consumers and buying yields less than that. If she charges \$9 then the will receive $2 \times 9 + 3 \times 1 = 21$. So \$9.

(b) Should S offer a price guarantee in April? Please explain. **[1]**

No. If she offers a price guarantee then she can charge \$10 now, but she will only receive \$20, which is less than the \$21 she receives in the scheme above.

(c) How would your answer to 1a change if the number for high value consumers in April were 4 instead of 8? **[2]**

She would set a price of \$4 in April selling everything for a profit of \$20.

2. What can you say about any two player extensive–form game that is finite, has perfect information, is strictly competitive, and the possible outcomes of which are 'Player 1 wins' and 'Player 2 wins?' **[1]**
That either Player 1 has a winning strategy or Player 2 has a winning strategy.

3. Imagine that I (the instructor, I, Player 1) am talking to you (the student, S, Player 2) about hiring you as an undergraduate grader for the next time I teach this course. Suppose that the status quo payoff vector is (1,1). Suppose further that if you were to take the job without pay then the payoff vector would be (10,0). Assume common knowledge and transferable utility.

   (a) Please explain the notion of transferable utility in your own words. **[1]**

   (b) Suppose that we agree that you will be payed 3. Then, using the standard bargaining solution, what is the value of your bargaining power? **[3]**
   Since the total surplus is 8 and you're getting 2, it is $1/4$

4. Consider the following two player four period alternating offer game. Two players are to divide \$1,000,000. In the spring Player 1 makes a proposal to split the money according to the fractions $(m_1, 1-m_1)$. If Player 2 accepts then the game is over and each player takes his/her money. If Player 2 rejects then the game is resumed in the summer and Player 2 proposes a split $(1-m_2, m_2)$. If Player 1 accepts this division of the money then the game is over and each player takes his/her money. Otherwise the game continues until the fall, at which time Player 1 gets to propose a split $(m_3, 1-m_3)$. If Player 2 accepts then the game is over. If he rejects then he gets to make the final proposal $(1-m_4, m_4)$ in the winter. If Player 1 rejects this last proposal then neither player receives anything. Assume standard discounting where both players value receiving \$2 now the same as receiving \$1 next season. Assume common knowledge and sequential rationality.

   (a) Please explain the concept of sequential rationality in your own words. **[1]**

   (b) How much money does Player 1 get and when? **[3]**
   \$625,000 in the spring. In the winter Player 2 would take everything for himself since in the winter this has become a standard ultimatum game. Hence in the fall, Player 1 would offer Player 2 $1-m_3 = 1/2$, which means that $m_3 = 1/2$. Knowing that Player 1 will get $1/2$ in the fall, Player 2 offers Player 1 $1-m_2 = 1/4$ in the summer, taking $m_2 = 3/4$ for himself. Knowing that Player 2 will get $3/4$ in the summer, Player 1 offers $1-m_1 = m_2/2 = 3/8$ in the spring, which means that $m_1 = 5/8$.

5. Consider a market in which every firm produces an identical good at a marginal production cost equal to 1. The market features an incumbent firm (Player 1) with capacity $q_1 = 5$. There are 8 consumers who are willing to pay at most \$6 each and will first buy from the firm posting the lowest price up to that firm's capacity, before purchasing from the other firm. If both firms charge the same price then they split the market evenly. Assume common knowledge and sequential rationality.

(a) Suppose first that Player 1 is a monopolist. How much profit would she make? **[1]**
She could sell 5 units at \$6 each which cost \$1 each to make, so \$25.

Now suppose that there is a potential entrant firm (Player 2) which can build a plant with capacity 5 at a cost of 5. The marginal production cost of Player 2 is also \$1.

(b) Suppose first that Player 2 has built his plant and that after construction of the plant both players simultaneously and independently choose a price (p_1 and p_2) using a mixed strategy. Please determine a mixed strategy Nash equilibrium for the pricing game. You may assume that the distribution F used is continuous. **[3]**
If a player charges \$6 then she will sell 3 units for a profit of \$15. So she will not charge less than $15/5 + 1 = \$4$. Player 2 wants to make Player 1 indifferent across prices between $[4, 6]$, i.e. Player 2 will choose F to ensure that

$$\forall p_1 \in [4, 6] : (p_1 - 1)\big[3F(p_1) + 5\{1 - F(p_1)\}\big] = 15.$$

Hence

$$F(p_1) = \begin{cases} 0, & p_1 < 4, \\ (5p_1 - 20)/(2p_1 - 2), & 4 \leq p_1 < 6, \\ 1, & 6 \leq p_1. \end{cases}$$

(c) *(tough bonus question; only attempt if you have time to spare)* Now suppose that both players can only set prices in quarters, e.g. they can charge \$6, \$5.75, \$5.50, etcetera, but not \$5.95. Suppose further that Player 1 commits to a price p_1^* *before* Player 2 decides whether or not to build his plant. So Player 1 first chooses p_1^*, then (having observed p_1^*) Player 2 decides whether or not to build a plant, and then Player 2 decides on a price p_2^*. Please identify any pure strategy subgame perfect Nash equilibria in this game. **[2]**
Suppose first that Player 1 sets $p_1^* > 4$. Then the best response for Player 2 is to enter and set $p_2^* = p_1^* - 0.25$. If Player 1 sets $p_1^* \leq 4$ then the best response for Player 2 is to stay out. So Player 2's best response is

$$\begin{cases} \text{enter}, & p_2^* = p_1^* - 0.25, & \text{if } p_1^* > 4, \\ \text{stay out}, & p_2^* \text{ arbitrary}, & \text{if } p_1^* \leq 4. \end{cases}$$

If Player 2 plays the above strategy then Player 1 cannot do better than choosing $p_1^* = 6$, which finishes the description of the subgame perfect Nash equilibrium. Please note that Player 1 receives the same payoff if she sets $p_1^* = 4$, but $p_1^* = 4$ is not a best response to Player 2 staying out.