Economics 402 Exam 5, May 6

You have one hour and fifteen minutes for this exam. Please do not use anything other than writing implements (pen, pencil, eraser), i.e. no electronics, calculator, etcetera. Please answer the questions on this exam paper itself. Please do not ask the proctor any questions to ensure that everyone can write their exam in a quiet disruption–free environment. You may leave when you're done. The number of points for each question is indicated in square brackets following the question.

Please show enough of your work to convince the grader that you know what you're doing.

Please assume common knowledge, sequential rationality, and the standard bargaining solution wherever applicable.

1. Please explain the concept *negotiation equilibrium.* **[2]**

2. Please consider a repeated game with $T = 2$ for which the stage game is as follows.

Stage game

1 \ 2	L	M	R
U	3,2	1,1	1,1
D	1,1	2,5	5,4

Assume there is no discounting (i.e. each player only cares about the sum of his/her payoffs over two periods).

(a) Please find a (pure strategy) subgame perfect equilibrium in the repeated game which does *not* specify a (pure strategy) Nash equilibrium in the stage game in the first period. Please establish that your answer is indeed a subgame perfect equilibrium. **[3]**

Play (D,R) in period 1; if (D,R) was played in period 1 then play (D,M) in period 2, else play (U,L). (D,M) and (U,L) are stage Nash profiles, so we only need to consider period 1. Player 1 does not gain by deviating in period 1. Player 2's total payoff from cooperating is $4 + 5 = 9$, whereas deviating gives him $5 + 2 = 7$, which is less.

(b) Now suppose that between periods 1 and 2, Player 2 can sell his goodwill to a Player 3 at a price p to be determined by negotiation between Players 2 and 3 at the time of sale. If Player 2 has bargaining power $2/3$ then what would be the value of p? **[2]**

$5 \times 2/3 = 10/3$

3. Please consider an infinitely repeated game with discount factor $0 < \delta < 1$, for which the stage game is as follows.

Stage game

1 \ 2	L	R
T	1,1	7,0
D	0,7	5,5

(a) For what values of δ is

Play (D,R) in every period until someone deviates, at which point (T,L) is played ever after

a subgame perfect equilibrium? **[3]**

Gain from deviating is 2. Loss from deviating is $4\delta + 4\delta^2 + \cdots = 4\delta/(1-\delta)$, so $\delta \geq 1/3$.

(b) **(hardest question on the exam)** For what values of δ is

Play (D,R) in odd periods and (T,R) in even periods until someone deviates, at which point (T,L) is played ever after

a subgame perfect equilibrium? **[3]**

Cooperating pays $5 + 5\delta^2 + 5\delta^4 + \cdots = 5/(1-\delta^2)$. Deviating pays $7 + \delta + \delta^2 + \cdots = 7 + \delta/(1-\delta)$. So $\delta \geq 2/3$.

(c) Please draw the set of equilibrium per–period payoffs that can arise if δ is close enough to one. Please label your axes and make sure it is clear where the corner points of your set are located. **[3]**

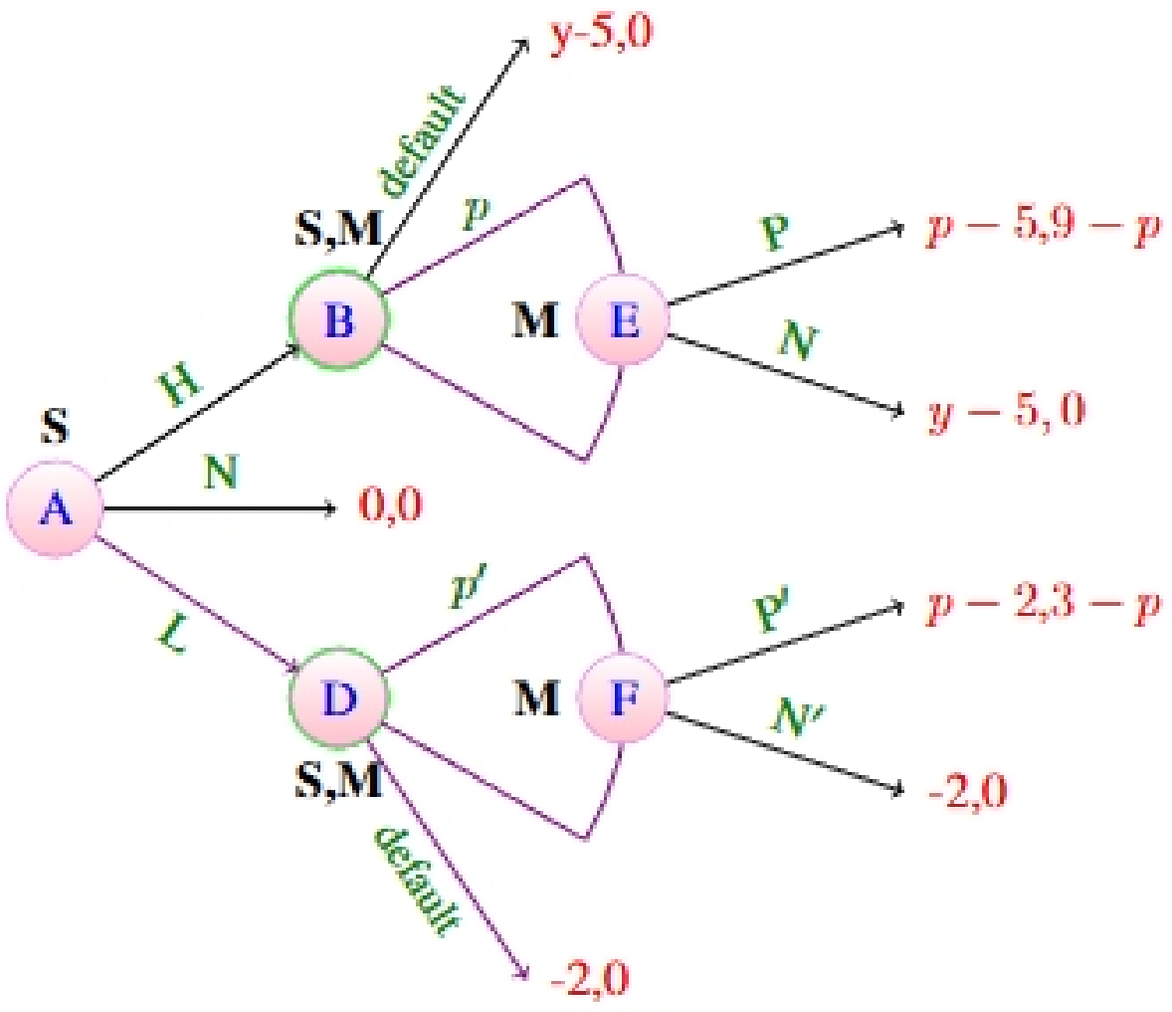
4. Please consider the game with the following time line:

$t = 1$: A scientist S chooses between making a high (H), a low (L), and no (N) investment in an idea at a cost of 5,2, and 0, respectively.

$t = 2$: If S chooses N then the game ends, otherwise S and a manufacturer M (who observes the investment S has made) negotiate a price p that M will pay S for the right to bring S's idea into production. The bargaining power of S is $1/3$. If S,M cannot come to an agreement then S can sell her idea to a third party at a price y if she chose H, but her idea has no residual value if she chose L.

$t = 3$: M decides whether (P) or not (N) to start production. If M produces then M will pay p to S, otherwise M pays S nothing. If M produces then M will receive 9 in revenue if S made a high investment and 3 if S made a low investment (minus p of course). If M decides not to produce then M receives nothing and pays S nothing, but S can sell her idea to a third party at the same price y mentioned above if she chose H, but her idea has no residual value if she chose L.

(a) Please draw the extensive form of this game. **[3]**



(b) Suppose that $y = 0$. Please determine the outcome of the negotiation equilibrium. **[3]**

Node E: M will choose P if $p \leq 9$

Node B: Since the surplus is 9, S will receive 3 and M will receive 6 (in addition to (-5,0)); the price would be 3

Node F: M will choose P' if $p \leq 3$

Node D: Since the surplus is 3, S will receive 1 and M will receive 2; the price would be 1