 Name____________________

You may use a pen/pencil and a 3"×5", handwritten note card on the exam. Answer all questions as completely as possible; show all of your work. If you run out of space on a problem, continue on the back of that sheet of paper. Good Luck!

Q1. (5 points each) Short and Simple.

(a) Write the complex number $3e^{i\pi}$ in the form $x + iy$.

(b) What is special about a normal-mode coordinate $Q_i(t)$?

(c) A solution to the 1D wave equation is $A\sin(kx)\sin(\omega t)$. What kind of wave is this?

(d) Describe, in words, what a dispersion relation is.

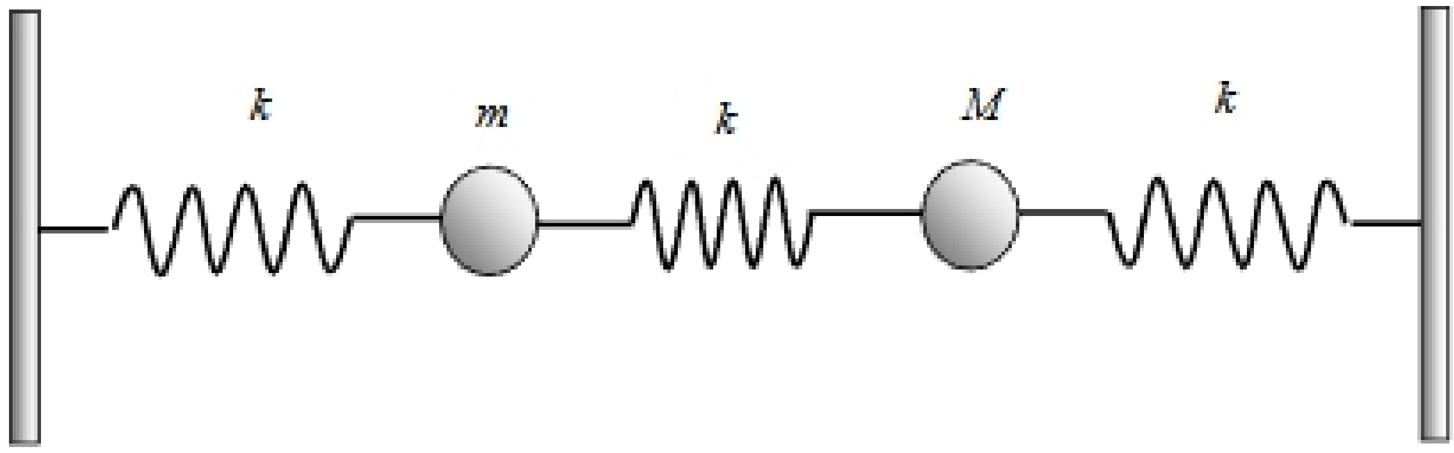
Q2. (10 points) Consider the potential energy function $V(q) = A(e^{q} + e^{-q} - 2)$. Find the harmonic approximation to this potential-energy function near $q = 0$. Identify the effective spring constant k.

Q3. (10 points) An harmonic oscillator with mass m and spring constant k has the following initial conditions: $q(0)=\alpha$, $\dot{q}(0)=0$. Write down the time-dependent motion $q(t)$ of this oscillator.

Q4. (10 points) Consider the equation $\begin{pmatrix} q_1(0) \\ q_2(0) \\ q_3(0) \end{pmatrix} = \left[\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} \alpha_1 + \begin{pmatrix} \sqrt{2} \\ 0 \\ -\sqrt{2} \end{pmatrix} \alpha_2 + \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix} \alpha_3 \right]$. Find α_3 in terms of the initial displacements $q_i(0)$.

Q5. (5 points) The dispersion relation for the coupled oscillator system can be written as $\omega(k) = 2\tilde{\omega}\sin\left(\frac{d}{2}k\right)$. Find the long wavelength limit of this expression and thus identify the parameter c^2 in the wave equation.

Q6. Consider the system illustrated in the following picture. The two objects have different masses, but all three springs are the same. Assume that the objects are constrained to move horizontally.



(a) (10 points) Write down the equation of motion for each object.

(b) (10 points) Find the characteristic equation that determines the normal-mode frequencies for this system. (Do not solve this equation.)