Math 128 Exam 3 Spring '10

You may use a (non-programmable) scientific calculator and a 3 × 5 note card for the exam.

This exam has 22 questions. The first 8 are True-False type questions worth two points each. The next 14 are multiple choice questions worth 6 points each.

Part I

Some of the statements below are always correct the others are sometimes incorrect. Indicate which are which by marking

$\boxed{\text{a}}$ for statements that are $\boxed{\text{A}}$lways true

$\boxed{\text{b}}$ for statements that may $\boxed{\text{B}}$e false

Yes **1.** If the maximum value of the function $f(x,y)$ is 0 then the minimum of the function $e^{-f(x,y)}$ is 1.

No **2.** Suppose profit, π, is a function of two variables, K, L and that you have selected the values of K and L which maximize π. The shadow price of K tells, approximately, how much more of input L you would need to maintain the same level of profit if the amount of input K were reduced one unit. see §14.2

No **3.** Given five data points $(x_1,y_1),\ldots,(x_5,y_5)$, the regression line for that set of data is the polygonal line connecting those five points.

The regression line is a certain straight line

No **4.** General theory insures that the function $f(x,y) = x^2 + xy^3 + e^{xy}$ will have a maximum value on the region of (x,y) with $x^2 + 4y^4 \leq 6$ and that that maximum will occur at a point where

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0.$$

this may not happen if max is on boundary

Yes **5.** The function f is defined for all (x,y). If at a critical point of $f(x,y)$ we have $f_{xx} = f_{yy} = 1$ and $f_{xy} = 0$ then that point is a local minimum for the function but it may not be the actual minimum.

Yes **6.** Suppose that you plan to use Lagrange multipliers to find the point on both of the surfaces

$$x^2 + 2y^2 = 10$$
$$x^2 - y^2 + z^2 = 100$$

which is closest to the point $(1,2,3)$. You will need to solve a system of 5 equations in 5 unknowns.

No **7.** The method of Lagrange multipliers produces candidate points for maximizing $f(x,y)$ given the constraint $g(x,y) = c$. The geometric idea underlying the method is that if $f(x_0,y_0) = A$ is the maximum then at the point (x_0,y_0) the curve $f(x,y) = A$ and the curve $g(x,y) = c$ cross at right angles.

the curves are parallel

8. The maximum value of $f(x) = x^a e^{-x}$, $x > 0$ is a number that depends on a. The Envelope Theorem can be used to study how that maximum depends on a.

Yes

Part II: Multiple Choice

9. Find the points, if any, where the function $f(x,y) = 12x^2 + 6y^2 + 12xy - 6x + 2$ might attain its maximum value and its minimum value.

- **a.** No maximum, no minimum.
- **b.** Maximum at $(1/2, -1/2)$, no minimum.
- **c.** No maximum, minimum at $(1/2, 1/2)$
- **d.** Maximum at $(1, 0)$, no minimum.
- $\boxed{\textbf{e.}}$ No maximum, minimum at $(1/2, -1/2)$
- **f.** Maximum at $(0, -1)$, minimum at $(1/2, 1/2)$
- **g.** No maximum, minimum at $(1/2, 0)$
- **h.** Maximum at $(1/2, -1/2)$, no minimum
- **i.** Maximum at $(1, -1/2)$, minimum at $(1, 1/2)$
- **j.** No maximum, minimum at $(1, 2)$

$f_x = 24x + 12y - 6$

$f_y = 12x + 12y$

$f_y = 0 \longrightarrow x = -y$

use with $f_x = 0$

get $x = \frac{1}{2}$, $y = -1/2$

$f_{xx} = 24$ $\quad f_{xy} = 12$

$f_{yy} = 12$

$f_{xx}f_{yy} - f_{xy}^2 > 0$

so possible min

10. Find the points, if any, where the function

$$f(x,y) = \frac{6x}{1+x^2} + 7y + 5$$

might attain its maximum value and its minimum value.

- $\boxed{\textbf{a.}}$ No maximum, no minimum.
- **b.** Maximum at $(-1/3, 1)$, no minimum.
- **c.** No maximum, minimum at $(-2, -4)$
- **d.** Maximum at $(1, 0)$, no minimum.
- **e.** No maximum, minimum at $(1/2, 1)$
- **f.** Maximum at $(0, -1)$, minimum at $(1/2, 1/3)$
- **g.** Maximum at $(1, 0)$, minimum at $(1/2, 0)$
- **h.** Maximum at $(1, 1/3)$, minimum at $(0, 0)$
- **i.** Maximum at $(1, -1/2)$, minimum at $(1, 1/2)$
- **j.** Maximum at $(0, -1)$, minimum at $(1, 2)$

$f_x = \frac{(1+x^2)6 - 6x(2x)}{(1+x^2)^2}$

$f_y = 7$

f_y never zero

no critical points

11. Find the points, if any, where the function $f(x,y) = 4x + 8y + 4xy + x^3 + 1$ might attain its maximum value and its minimum value.

$\boxed{\text{a.}}$ No maximum, no minimum.
b. Maximum at $(-2,1)$, no minimum.
c. No maximum, minimum at $(2,4)$
d. Maximum at $(-2,4)$, no minimum.
e. No maximum, minimum at $(1,1)$
f. Maximum at $(0,-1)$, minimum at $(4,-2)$
g. Maximum at $(1,0)$, minimum at $(0,0)$
h. Maximum at $(0,0)$, minimum at $(1,0)$
i. Maximum at $(1,-4)$, minimum at $(-4,2)$
j. Maximum at $(0,4)$, minimum at $(1,2)$

$f_x = 4 + 4y + 3x^2$
$f_y = 8 + 4x$
$f_y = 0 \rightarrow x = -2$
That and $f_x = 0$
$4y + 16 = 0$
$y = -4$
at ~~the~~ that point
$f_{xx} = 6x = -12$
$f_{yy} = 0$
} saddle pt.

12. The function $f(x,y) = x^2 + 2xy^2 + 2y^2$ has three critical points. Locate them and classify the type of critical points they are. When listed from top to bottom (i.e., in order of decreasing y coordinate) they are

a. A saddle point, a local maximum, a local minimum.
b. A saddle point, a local minimum, a local maximum.
c. A local minimum, a local minimum, a local minimum.
d. A local maximum, a saddle point, a local minimum..
e. A local minimum, a saddle point, a local maximum.
f. A saddle point, a local minimum, a saddle point.
g. A saddle point, a saddle point, a saddle point.
h. A local maximum, a local minimum, a local minimum.
i. A local maximum, a local maximum, a local maximum.
j. A local maximum, a local minimum, a saddle point.

$f_x = 2x + 2y^2$
$f_y = 2xy + 4y$
$f_y = 0 \rightarrow y(2x+2) = 0$
so $y = 0$
+ $f_x = 0$ gives $x = 0$
or $x = -1$ in which
case $y^2 = 1$
$y = \pm 1$

so $(-1,1)$
$(0,0)$
$(-1,-1)$

$f_{xx} = 2$
$f_{yy} = 2x + 4$
$f_{xy} = 4y$

2^{nd} deriv test
saddle, min, saddle