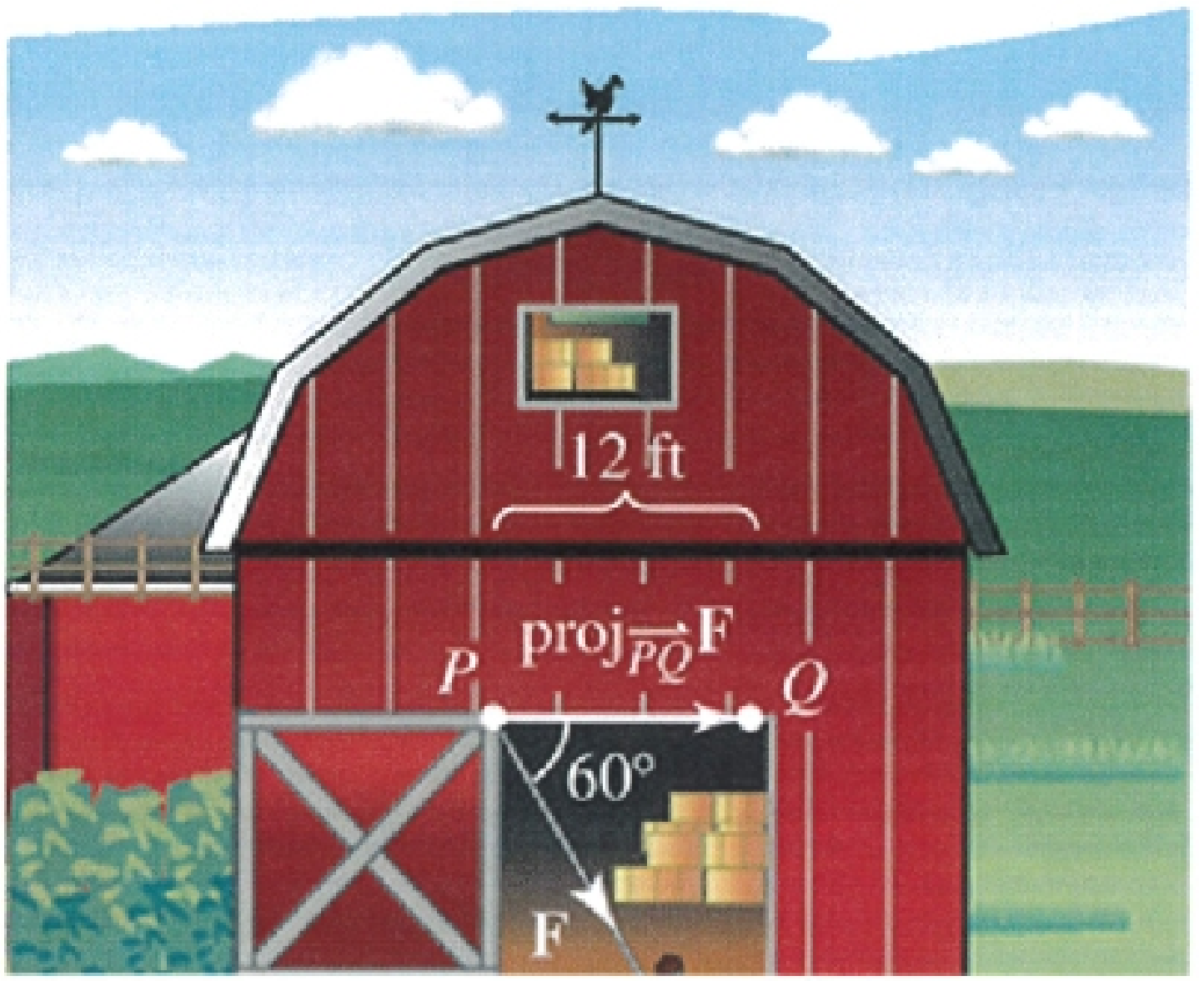
1

To close a sliding door, a person pulls a rope with a constant force of 50 pounds at a constant angle of $60°$. Find the work done in moving the door 12 feet to its closed position.



$W = F \cos\theta \, d$ $\quad 50\cos(60)\,12 = \underline{300 \text{ lb ft}}$

(2)

Let C be the curve represented by the parametric curve

$$\vec{r}\,(t) = 2e^t\hat{i} + e^t\cos(t)\hat{j} + e^t\sin(t)\hat{k}.$$

Give the equation of the plane generated by the unit tangent vector $\vec{T}\,(t_0)$ and the unit normal vector $\vec{N}\,(t_0)$.

0/3

#2

$$\vec{r}(t) = 2e^t\hat{i} + e^t\cos(t)\hat{j} + e^t\sin(t)\hat{k}$$

$$T(t) = \frac{r'(t)}{|r'(t)|} \qquad N(t) = \frac{T'(t)}{|T'(t)|}$$

$$r'(t) = 2e^t\hat{i} + (e^t\cos(t)\hat{j} - e^t\sin(t)\hat{j}) + e^t\sin(t)\hat{k} + e^t\cos(t)\hat{k}$$

$$|r'(t)| = \sqrt{(2e^t)^2\hat{i} + [e^t\cos(t) - e^t\sin(t)]^2 + [e^t\sin(t) + e^t\cos(t)]^2}$$

$$4e^{2t}\hat{i} + [e^{2t}\cos^2(t) + e^{2t}\sin^2(t) - e^{2t}\sin(2t)]\,[e^{2t}\sin^2(t) + e^{2t}\cos^2(t) + 2e^{2t}\cos(t)\sin(t)]$$

$$4e^{2t}\hat{i} + 2e^{2t} - e^{2t}\sin(2t) + e^{2t}\sin(2t) + e^{2t}$$

$$4e^{2t} + e^{2t} + e^{2t} = \sqrt{6e^{2t}} = \sqrt{6}\,e^t$$

$$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{2e^t}{\sqrt{6}e^t}\hat{i} + \frac{e^t\cos(t) - e^t\sin(t)}{\sqrt{6}e^t}\hat{j} + \frac{e^t\sin(t) + e^t\cos(t)}{\sqrt{6}e^t}\hat{k}$$

$$\boxed{T(t_0) = \frac{2}{\sqrt{6}}\hat{i} + \frac{\cos(t_0) - \sin(t_0)}{\sqrt{6}}\hat{j} + \frac{\sin(t) + \cos(t)}{\sqrt{6}}\hat{k}}$$

$$N(t) = \frac{T'(t)}{|T(t)|}$$

$$T'(t) = 0\hat{i} + \frac{-\sin(t) - \cos(t)}{\sqrt{6}}\hat{j} + \frac{\cos(t) - \sin(t)}{\sqrt{6}}\hat{k}$$

$$* = 0\hat{i} - \frac{\sin(t) + \cos(t)}{\sqrt{6}}\hat{j} + \frac{\cos(t) - \sin(t)}{\sqrt{6}}\hat{k}$$

$$|T(t)| = \sqrt{0^2 + \left(-\frac{\sin(t) + \cos(t)}{\sqrt{6}}\right)^2 + \left(\frac{\cos(t) - \sin(t)}{\sqrt{6}}\right)^2}$$

$$\frac{\sin^2(t) + \cos^2(t) + 2\sin(t)\cos(t)}{6} + \frac{(\cos^2(t) + \sin^2(t) - 2\sin(t)\cos(t))}{6}$$

$$= \sqrt{\frac{\sin^2(t) + \cos^2(t) + 2\sin(t)\cos(t) + \cos^2(t) + \sin^2(t) - 2\sin(t)\cos(t)}{6}}$$

$$|T(t)| = \sqrt{\frac{2}{6}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

$$N(t_0) = 0\hat{i} + \frac{-\sin(t) + \cos(t)}{\sqrt{6}\cdot\sqrt{2}}\cdot\sqrt{3}\hat{j} + \frac{\cos(t) - \sin(t)}{\sqrt{6}\cdot\sqrt{2}}\cdot\sqrt{3}\hat{k}$$

$$\boxed{N(t_0) = 0\hat{i} - \frac{\sin(t) + \cos(t)}{\sqrt{2}}\hat{j} + \frac{\cos(t) - \sin(t)}{\sqrt{2}}\hat{k}}$$