MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2014**

Practice Problem Set 11

Text: Dourmashkin, Belcher, and Liao; Introduction to E & M MIT 8.02 Course Notes Revised.

Week Fourteen Interference and Diffraction	**Problem Set 10 Due Tuesday May 6 at 9 pm**
W14D1 M/T May 5/6	Maxwell's Equations and One Dimensional Wave Equation
Reading	Course Notes: Sections 13.4, 13.6-13.7
W14D2 W/R May 7/8	Energy Flow and the Poynting Vector Polarization Expt 5 MW; Interference
Reading	Course Notes: Sections 13.8, 13.10, 14.1-14.3
W14D3 F May 9	PS10 Maxwell's Equations, Energy Flow and the Poynting Vector
Reading	Course Notes: Section 13.5, 13.6-7, 13.11, 13.12

Week Fifteen Poynting Vector and Energy Flow; Final Review

W15D1 M/T May 12/13	Diffraction; Expt. 6: Interference and Diffraction
Reading	Course Notes: Sections 14.4-14.11
W15D2 W/R May 14/15	Final Review

Monday May 19 9 am-12 noon in Johnson Athletic Center Second Floor

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Problem 1: Electromagnetic Waves and the Poynting Vector

We have been studying one particular class of electric and magnetic fields solutions called *plane sinusoidal traveling waves*. One special example is an electromagnetic wave traveling in the positive x-direction with the speed of light c is described by the functions

$$\vec{\mathbf{E}}(x,y,z,t) = E_y(x,t)\hat{\mathbf{j}} = E_{y,0}\sin\left(\frac{2\pi}{\lambda}(x-ct)\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}}(x,y,z,t) = B_z(x,t)\hat{\mathbf{k}} = \frac{1}{c}E_{y,0}\sin\left(\frac{2\pi}{\lambda}(x-ct)\right)\hat{\mathbf{k}}$$

Suppose the plane wave front (infinite yz-plane) traveling at speed c in the positive x-direction passes through a rectangular volume of space that has area A perpendicular to the direction of propagation and length $c\Delta t$, corresponding to the distance the electromagnetic wave travels in time Δt. Inside this volume of space the electric and magnetic fields store energy and that energy changes in time as the wave passes through the volume. The energy per time per area transported by this electromagnetic wave is called the *Poynting vector* and is defined as

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}.$$

The power transmitted by the Poynting vector through a surface is the flux given by the expression

$$P(t) = \iint_{surface} \vec{\mathbf{S}}(t)\cdot\hat{\mathbf{n}}\, da.$$

The flux of the Poynting vector on a surface describes the energy that 'flows' across the surface.

a) What the Poytning vector associated with this wave?

b) What is the time-averaged Poynting vector field on the fixed plane $x = 0$ over one period? The definition of a time average of a periodic function $f(t)$ over one period is

$$\langle f(t)\rangle = \frac{1}{T}\int_0^T f(t)\,dt$$

Recall that $c = 1/\sqrt{\mu_0\varepsilon_0}$.

c) What is the **time-average** (over one period) of the **energy density** stored in the electric and magnetic fields at the point (x, y, z, t). Recall that $u_{elec} = (1/2)\varepsilon_0 E^2$ and $u_{mag} = B^2 / 2\mu_0$. How is that related to the magnitude of the time-averaged Poynting vector?

d) What is the **time-averaged energy** stored in the electric and magnetic fields in a rectangular volume of cross-sectional area A and length $c\Delta t$, where the length $c\Delta t$ is aligned along the x-axis and $c\Delta t << \lambda$. The last assumption allows us to assume that $\left|\vec{S}(x,t)\right|$ is nearly uniform across the box.

e) What is the time-averaged rate of change of the total energy stored in the electric and magnetic fields in the rectangular volume of cross-sectional area A and length $c\Delta t$?

f) What is the time-averaged flow of power through the surface of area A located on the surface given by $x = x_p$, perpendicular to the direction of propagation? How does the power that through the rectangular surface (this flux is negative since it flows into the volume) compare to the time derivative of the energy stored in the fields inside the volume?