

First Examination
Physics 5013, Mathematical Methods of
Physics

October 20, 2006

Instructions: Attempt all parts of this exam. If you get stuck on one part, assume an answer and proceed on. Do not hesitate to ask questions. Remember this is a closed book, closed notes, exam. *Good luck!*

1. In this problem you are to evaluate the definite integral

$$I = \int_0^{\infty} \frac{dx}{x} \frac{\sin x}{(x^2 + a^2)^2}, \quad a > 0,$$

using the residue theorem. Do this as follows:

- (a) Show that I is part of a closed contour integral. Specify precisely what the integrand and the contour of integration is.
 - (b) Evaluate the various parts of the contour integral. Use Jordan's lemma if appropriate.
 - (c) Evaluate the entire contour integral by the residue theorem.
 - (d) Finally give the value of I as a function of a .
2. The tangent function may be expressed as a Taylor series about the origin:

$$\tan z = \sum_{n=0}^{\infty} a_n z^n.$$

- (a) Which coefficients a_n are zero? Why?

- (b) Calculate the first three nonzero coefficients using any method you like.
- (c) What is the radius of convergence of this power series? Why?
- (d) Calculate the ratio of adjacent nonzero coefficients for large n in terms of this radius of convergence.
- (e) How well is this asymptotic ratio satisfied for the first three nonzero coefficients that you calculated in part 2b? [It turns out that the ratio of the k th to $k + 1$ th nonzero coefficients agrees with the asymptotic limit to k decimal places!]