

Exchange Property

If $f_i > f_j$ and $d(a_i) > d(a_j)$, then exchanging a_i and a_j would make the objective function value decrease in

$$\begin{aligned} & (d(a_i) \cdot f_i + d(a_j) \cdot f_j) - (d'(a_i) \cdot f_i + d'(a_j) \cdot f_j) \\ &= (d(a_i) \cdot f_i + d(a_j) \cdot f_j) - (d(a_j) \cdot f_i + d(a_i) \cdot f_j) \\ &= (d(a_i) - d(a_j))(f_i - f_j) \end{aligned}$$

Self-reducible Optimality



If we assign the weight of each internal node with the total weight of its descendant leaves, then removal a subtree results still in an optimal tree for remainder's leaves.

