

## LESSON #30 HANDWRITTEN PRACTICE EXERCISE KEY:

### 10.9

#### Part 1:

1. If an object is left of b then b is right of that object.
2. Every small object in back of c is a dodec.
3. Every cube that is not b is either larger or smaller than b.
4. If d is a dodec, then anything that is d is a dodec.
5. If a is smaller than something that is itself smaller than b, then a is smaller than b.
6. Anything that is larger than c is not c.
7. Every object is between a and d or is not between a and d.
8. Every object is between a and d or is not between d and a.
9. Every dodec is d or small.
10. If every cube is left of e, then there isn't any cube that is not left of e.

#### Part 2:

You should be able to falsify and build counterexample worlds for sentences #2, #3, #9.

#### Part 3:

- 4      cf. FO checking form:  $R(d) \rightarrow \forall x (x = d \rightarrow R(x))$   
          can still see the necessity: "If d has property R then any object that is d has property R"
- 7      cf. FO checking form:  $\forall x (R(x, a, d) \vee \neg R(x, a, d))$   
          can still see the necessity: "Each object is either in R-relation to a and d or else it is not in R-relation to a and d"
- 10     cf. FO checking form:  $\forall x (R(x) \rightarrow S(x, e)) \rightarrow \neg \exists y (R(y) \wedge \neg S(y, e))$   
          can still see the necessity: "If every object that has property R stands in S-relation to e, then there is no object that has property R and that does not stand in S-relation to e"

#### Part 4:

- 1-- If any object is redder than b, then b eats the object. [*just pick any two relations that are not symmetrical*]
- 5-- If a kills something that kills b, then a kills b. [*i.e., just pick a predicate that is not transitive*]
- 6-- Anything that lives in the same house as c is not c. [*i.e., just pick a predicate that doesn't inherently rule out an identity relation*]
- 8-- Every object helps a to eat d or else does not help d to eat a. [*i.e., just pick a predicate that is not symmetrical*]

### 10.10

(answer **b** - FO consequence in the argument)

Truth-functional form of argument:

A ∧ B	(replace each atomic sentence)
C ∧ D	(replace each atomic sentence)
—E—	(replace entire quantified phrase)

RESULT: NO CONSEQUENCE RELATION APPARENT WITH TRUTH-FUNCTIONAL FORM, SO THERE IS **NOT** TAUTOLOGICAL CONSEQUENCE HERE.

Replacement method using nonsense predicates:

Morg(a) ∧ Morg(b)	(replace just predicates with nonsense predicates)
Gord(a) ∧ Jorm(b)	(replace just predicates with nonsense predicates)
—∃x (Morg(x) ∧ Gord(x)) ∧ ∃x (Morg(x) ∧ Jorm(x))—	(replace just predicates with nonsense predicates)

RESULT: YES, CAN SEE A CONSEQUENCE RELATION EVEN WITH PREDICATES REPLACED BY NONSENSE PREDICATES, SO THERE **IS** FO CONSEQUENCE HERE

### 10.11

(answer **c** - logical consequence but **not** FO- or tautological consequence in the argument)

Truth-functional form of argument:

A ∧ B	(same replacement method described above)
B ∧ C	
—D—	

RESULT: AGAIN, NO CONSEQUENCE RELATION APPARENT WITH TRUTH-FUNCTIONAL FORM, SO THERE IS **NOT** TAUTOLOGICAL CONSEQUENCE HERE

Replacement method using nonsense predicates:

Morg(a) ∧ Morg(b)
Gord(a) ∧ Jorm(b)
—∃x (Morg(x) ∧ Jorm(x) ∧ ¬Freug(x,x))—

RESULT: THE FINAL CONJUNCT OF THE CONCLUSION DOESN'T CLEARLY FOLLOW, SO THERE IS **NO** FO CONSEQUENCE RELATION APPARENT HERE EITHER. THIS FACT IS CONFIRMED MORE CLEARLY WITH THE FOLLOWING FO COUNTEREXAMPLE, IN WHICH THE CONCLUSION IS CLEARLY FALSIFIED IF WE REPLACE THAT FINAL CONJUNCT PREDICATE WITH A BINARY PREDICATE WHOSE NEGATION YIELDS A CONTRADICTION, "NOTHING IS THE SAME HEIGHT AS ITSELF":

FO COUNTEREXAMPLE:

	Ugly(a) $\wedge$ Ugly(b)
	Hungry(a) $\wedge$ Dead(b)
	$\exists x$ (Ugly(x) $\wedge$ Dead(x) $\wedge$ $\neg$ SameHeight(x,x))

THE POINT IS THAT IF THE CONCLUSION REALLY WERE A FO-CONSEQUENCE OF ITS PREMISES (WHICH IT'S NOT), IT WOULDN'T MATTER THAT WE REPLACED THE PREDICATES IN THIS WAY, BECAUSE FO CONSEQUENCE DOESN'T DEPEND ON THE MEANINGS OF THE PREDICATES INVOLVED (OTHER THAN THE IDENTITY PREDICATE).

SO, THE ONLY CONSEQUENCE PRESENT IN THE ORIGINAL ARGUMENT ABOVE FOR 10.11 IS PLAIN LOGICAL CONSEQUENCE, NOT FO- OR TAUTOLOGICAL CONSEQUENCE.

### 10.12

(answer **b** - FO consequence)

Truth-functional form of argument:

	A $\rightarrow$ B
	$\neg$ B
	C

RESULT: NO CONSEQUENCE RELATION IS APPARENT WHEN ARGUMENT IS CONVERTED TO TRUTH-FUNCTIONAL FORM, PROVING THAT THERE IS NO TAUTOLOGICAL CONSEQUENCE HERE.

Replacement method using nonsense predicates:

	$\forall x$ Mag(x) $\rightarrow$ $\exists y$ Jag(y)	(translate this line: "If everything has property Mag, then something has property Jag")
	$\neg \exists y$ Jag(y)	(translate this line: "Nothing has property Jag")
	$\exists x \neg$ Mag(x)	(translate: "Something does not have property Mag")

RESULT: YES, CAN SEE A CONSEQUENCE RELATION EVEN WITH PREDICATES REPLACED BY NONSENSE PREDICATES, SO THERE **IS** FO CONSEQUENCE HERE