PRACTICE EXERCISE ANSWER KEY FOR LESSON #10

(Discussion of answers to exercise 4.8 -- pp. 104-105 of textbook)

I apologize in advance for the length of the discussions of some of the harder sentences below. There's simply no 'easy' way to do this. I've tried, however, to make the discussions as smooth and readable as possible, so sit down in your most comfortable chair, allow yourself an undisturbed half hour or so to carefully read and think through this, and with luck you'll come out understanding the basic concepts of possibility, necessity, and tautological necessity. As you read through the answer explanations below, you should consult as well **slide #9** of the **PowerPoint** presentation for **lesson 08** posted on Blackboard, where there is a graphical answer-key for these 10 sentences showing where each of them falls in the various circles of the Euler diagram representing possibility and necessity on p. 102 of the textbook.

1. a = b

   logically possible

   This one is pretty straightforward: It states that there is one object named by both 'a' and 'b' (i.e., 'a' and 'b' are identical; one and the same object). Though this arrangement certainly may be the case in some worlds, we can also imagine many worlds in which 'a' and 'b' would refer to two *distinct* objects instead of one and the same object. Therefore, this sentence is merely logically-possible (or TW-possible, since we're talking about a sentence that can describe worlds in the Tarski's World program) and *not* logically necessary (i.e., the sentence doesn't have to be true in all worlds).

2. a = b v b = b

   logically necessary

   This sentence is a disjunction of two disjuncts: one being an atomic sentence (**a = b**) that by itself would be merely logically possible (i.e., the same atomic sentence just discussed under #1 above) and the other being a different atomic sentence (**b = b**) that by itself is necessarily true in all worlds (i.e., because it expresses the principle of self-identity that any object is necessarily identical to itself). Because *only one disjunct* of a larger disjunction needs to be true in order for the entire disjunction to be true (recall the truth-table for disjunction) and because the second disjunct (**b = b**) in this sentence is a logical truth and will *always* be true, therefore *the entire disjunction will always be true* in every world, making it in turn a logical truth (logically necessary).

3. a = b ∧ b = b

logically possible

The only difference between the sentence here in #3 and the one in #2 is, of course, that #3 is a *conjunction* instead of a disjunction. But that makes all the difference, because for the entire *conjunction* to be true in some world, *each and every one of its conjuncts* (i.e., the atomic sentences that make it up) must be individually true in that world. So, the sentence here in #3 will be true only in worlds where the first conjunct **a = b** is true, which, as we saw in the discussion of #1 above, will be the case in *some* worlds but *not* all worlds. (The fact that the second conjunct, **b = b**, will be true in every world is irrelevant here because the entire conjunction won't be true unless the first conjunct, **a = b**, is *also* true, but as I just explained, that won't be the case in every world.) Given, then, that the entire conjunction will be true in some worlds (where **a = b** is true) but not in other worlds (where **a = b** is false), the entire conjunction is merely logically *possible*, not necessary.

4. ¬(Large(a) ∧ Large(b) ∧ Adjoins(a,b))

TW-necessary

This sentence is necessarily true in every world you can build with Tarski's World only because of *a peculiarity of Tarski's World itself*, namely, the restriction built into the software that large objects may not adjoin each other. Of course, this requirement isn't otherwise any sort of *logical* necessity (i.e., we can easily imagine an alternative version of Tarski's World or some other software that would contain no restriction against large objects adjoining), so it is a *TW-necessity*, not a normal logical necessity.

5. Larger(a,b) ∨ ¬Larger(a,b)

tautologically necessary (i.e., a tautology)

This sentence says that 'a' *is* larger than 'b', or (else) 'a' *isn't* larger than 'b'—and given that any sentence of FOL has one of only two truth-values, true or false, then there's no escaping the fact that this sentence will necessarily be true in every world (i.e., it either *is* or it *isn't* the case that 'a' is larger that 'b'; there is no third option). Notice that the atomic sentences making up each of the disjuncts are exactly the same (**Larger(a,b)**), so we can test more specifically for *tautological* necessity by replacing each occurrence of this atomic sentence with the same capital letter 'A' so as to 'blind' ourselves to any meaning-content that is internal to the atomic sentences. We make the substitution like this: **A ∨ ¬A** Notice that when we do this, we can still 'see' the necessity involved in the overall sentence (i.e., that this sentence *must* be true in any world) even though the only aspect of the sentence that we can still 'see' is the work that the Boolean connectives **∨**

and ¬ are doing to structure the overall sentence. It turns out that this sentence is a case of what classical logicians called the "law of the excluded middle", and it is probably the most famous tautology of all.

6. Larger(a,b) v Smaller(a,b)

logically possible

Given two objects 'a' and 'b', this disjunction expresses two possibilities about their size-relation to each other: (1) 'a' is larger than 'b' or, conversely, (2) 'a' is smaller than 'b'. Either of those options are possibly realized in some worlds, no doubt, but notice that there is a third option (3) not addressed by this sentence: 'a' and 'b' might be the *same size*. In such a case, the sentence here will be false. So then, the sentence will sometimes be true (in worlds instantiating either of the first two options) but sometimes false (in worlds instantiating the third option). The sentence is only *possibly* true, *not* necessarily true.

7. ¬Tet(a) v ¬Cube(b) v a ≠ b

logically necessary

We discussed this one in class. Just trying to read and understand it can be headache-inducing, so you'll probably have to approach it like a puzzle to solve. Notice that the entire sentence is a *disjunction*, meaning that it will be *true* in any world where *one or more* of the disjuncts is individually true (recall the truth-table for disjunction). But by the same token, the entire disjunction will only be *false* in a world where *all three* of the disjuncts are *each false* (all three false at the same time). This latter fact suggests a strategy for how we can figure out whether this sentence is a logical possibility or else a necessity: If we can imagine a world in which all three disjuncts are *false at the same time*, we will have proven that this sentence is (at most) merely a logical *possibility* (because we will have, in effect, constructed a counterexample world, showing that the sentence as a whole *can* be made false). If, on the other hand, we can't construct such a world (where all three disjuncts are false at the same time), then we'll have good evidence that this sentence is a logical necessity.

Well, when we try out this strategy, we discover that we *cannot* falsify all three disjuncts together at the same time. Working backwards from right to left, when we make the last disjunct **a ≠ b** to be *false*, we get its opposite: **a = b**. That means that in the world we are imagining, 'a' and 'b' are two names for one and the same object. Okay, so far so good. Now, we falsify the second disjunct **¬Cube(b)** and that gives us its opposite, **Cube(b)**, which says that in the world we are imagining, 'b' is a cube. Alright, that's still okay … this world we're imagining so far contains an object with two names, 'a' and 'b', and this object is a cube. But now we run into problems when we try to falsify the remaining disjunct **¬Tet(a)**, for if we assume its opposite,