

Econ 311: Definitions of Exogeneity and Causality

- (1) Weak Exogeneity
- (2) Granger Noncausality
- (3) Strong Exogeneity
- (4) Super Exogeneity
- (5) Strict Exogeneity
- (6) External Variable
- (7) Policy Invariant Structural Parameter
- (8) Autonomous Relationship

Take the Parametric Case

$$Q_t = (Y_t, X_t) \quad t = 1, \dots, T$$

$\theta \in \Theta \subset R^n$ parametric family

$$1. \quad f(Q_t | Q_{t-1}, \theta) = f(Y_t | X_t, Q_{t-1}, \theta) \cdot f(X_t | Q_{t-1}, \theta)$$

$$\varphi = f(\theta) \quad 1 - 1$$

$$\varphi \in \Phi; \theta \in \Theta$$

Can always write (with φ_1, φ_2 together as reparameterization of θ)

$$f(Q_t | Q_{t-1}, \theta) = f(Y_t | X_t, Q_{t-1}, \varphi_1) f(X_t | Q_{t-1}, \varphi_2)$$

Weak Exogeneity

Notice we can always achieve (1).

Suppose we seek to learn about ψ and suppose that

$$(a) \psi = g(\varphi_1) \quad g \text{ is not necessarily}$$

1-1;

(b) φ_1 and φ_2 are variation free:

$$(\varphi_1, \varphi_2) \in \Phi_1 \times \Phi_2 = \{(\varphi_1, \varphi_2) : \varphi_1 \in \Phi_1, \varphi_2 \in \Phi_2\}$$

Definition: Then X_t is weakly exogenous (for ψ) if we have (a) and (b) holding.