Lecture: 25-8-14 Professor Shabonskaya Calculus 1760-~~00~~

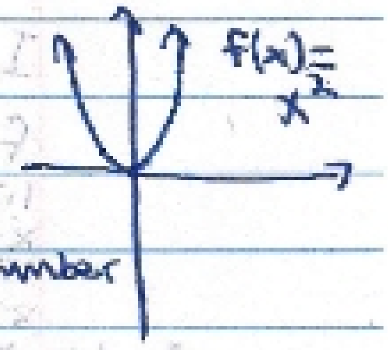
Section 5.8 Antiderivatives

Antiderivative Problem: Given a function f

The function F (F is an antiderivative of f) for x in the interval ~~of~~ I if $F'(x) = f(x)$ for x in the interval I. $\left(\frac{d}{dx} F(x) = f(x)\right)$

where $I = \mathbb{R}, (a,b), [a,b), (a,b], (-\infty, a), (-\infty, a], (a, \infty), [a, \infty)$

Ex: ① Find an antiderivative of $f(x) = x^2$



Solution: $F(x) = x^3/3$, $F(x) = \frac{x^3}{3} + C$

Hint: Power Rule for derivatives: $\frac{d}{dx}(x^r) = rx^{r-1}$ r is any real number

~~F'(x) = x~~

Suggestion 1: $F(x) = \frac{x^3}{3}$ (check to see if correct)

$\left(\frac{x^3}{3}\right)' = \left(\frac{1}{3}x^3\right)' = \frac{1}{3}(x^3)' = \frac{1}{3}(3x^2) = x^2$

because $f(x) = x^2$, $\boxed{F(x) = x^3/3}$

Question: Is $F(x) = x^3/3$ is it the only antiderivative of $f(x) = x^2$

Suggestion 2: $F(x) = \frac{x^3}{3} + 1$ is an antiderivative of $f(x) = x^2$

because $\left(\frac{x^3}{3} + 1\right)' = \left(\frac{x^3}{3}\right)' + (1)' = \boxed{x^2}$ $\boxed{F(x) = \frac{x^3}{3} + 1}$

Therefore we know all antiderivatives of $f(x) = x^2$, which are $F(x) = \frac{x^3}{3} + C$, (c is a constant).

Definition of an antiderivative problem: $F(x) + C$ (c is a constant). These are all antiderivatives, also known as a general antiderivative of $f(x)$, x is in the interval I. In this context $F(x)$ is a particular antiderivative of $f(x)$. We know $F(x) + C$ is an antiderivative, ~~because F(x) is an antiderivative~~ because $(F(x)+C)' = F'(x) = f$ is an antiderivative

② Find the general (all antiderivatives) antiderivative of $f(x) = 2e^{2x}$ $\quad (e^{ax})' = e^{ax}\cdot(a)$

Solution: $F(x) = e^{2x} + C$

$F(x) = e^{2x}$ is a particular antiderivative because $(e^{2x})' = 2e^{2x}$

$F(x) = e^{2x} + C$, c is ~~a constant~~ the general antiderivative of $f(x) = 2e^{2x}$