$y''' - 9y'' + 23y' - 15y = e^x$

$r^3 - 9r^2 + 23 - 15 = 0$

$r = \frac{p}{q} = \frac{\pm 1, 3, 5, 15}{1}$

$y = c_1 e^x + c_2 e^{3x} + c_3 e^{5x} + Y$

Undetermine Coefficient

$Y = Axe^x$

$Y' = Axe^x + Ae^x$

$Y'' = Axe^x + 2Ae^x$

$Y''' = Axe^x + 3Ae^x$

$3Ae^x + Axe^x - 9Axe^x - 18Ae^x + 23Axe^x + 23Ae^x - 15Axe^x = e^x$

$8A = 1 \quad A = 1/8$

$Y = 1/8 x e^x$

$y = c_1 e^x + c_2 e^{3x} + c_3 e^{5x} + 1/8 x e^x$

$$W = \begin{vmatrix} e^x & e^{3x} & e^{5x} \\ e^x & 3e^{3x} & 5e^{5x} \\ e^x & 9e^{3x} & 25e^{5x} \end{vmatrix}$$

$e^x(e^{3x}5e^{5x} - e^{5x}3e^{3x}) - 9e^{3x}(e^x 5e^{5x} - e^{5x}e^x) +$

$25e^{6x}(e^x 3e^{3x} - e^{3x}e^x)$

$e^x(2e^{8x}) - 9e^{3x}(4e^{6x}) + 25e^{5x}(2e^{4x})$

$2e^{9x} - 36e^{9x} + 50e^{9x} =$

$W = \boxed{16e^{9x}}$

Variation of Parameters

$Y = u_1 y_1 + u_2 y_2 + u_3 y_3$

$u_n = \int \frac{W_n \, g(x)}{W}$

$$W_1 \begin{vmatrix} 0 & e^{3x} & e^{5x} \\ 0 & 3e^{3x} & 5e^{5x} \\ 1 & 9e^{3x} & 25e^{5x} \end{vmatrix} \quad 1(2e^{8x})$$

$u_1 = \int \frac{W_1 5x}{W} = \int \frac{2e^{8x}e^x}{16e^{9x}} = \int 1/8 = 1/8 x$

$$W_2 \begin{vmatrix} e^x & 0 & e^{5x} \\ e^x & 0 & 5e^{5x} \\ e^x & 1 & 25e^{5x} \end{vmatrix} \quad -1(9e^{6x}) = -9e^{6x}$$

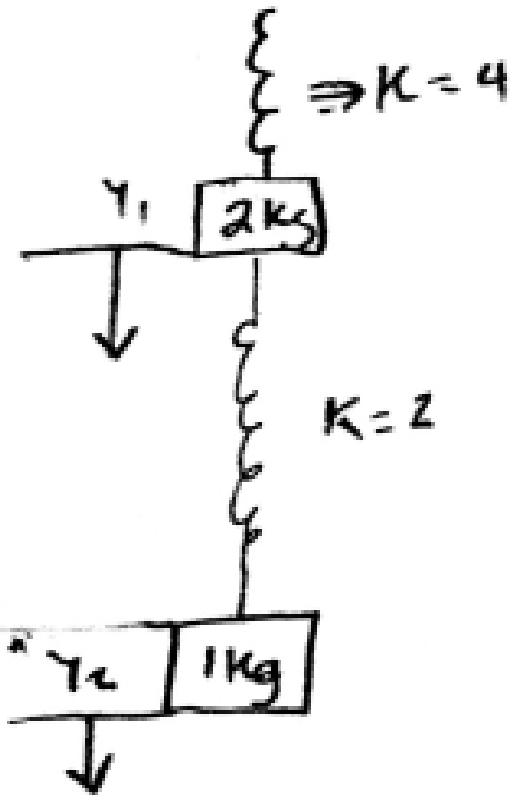
$u_2 = -\int \frac{4e^{6x}e^x}{16e^{9x}} = -1/4\left(e^{-2x} \quad 1/8 e^{-2x}\right.$

$$W_3 \begin{vmatrix} e^x & e^{3x} & 0 \\ e^x & 3e^{3x} & 0 \\ e^x & 9e^{5x} & 1 \end{vmatrix} \quad 1(2e^{4x}) = 2e^{4x}$$

$u_3 = \int \frac{2e^{4x}e^x}{16e^{9x}} = 1/8\left(e^{-4x} = 1/32 e^{-4x}\right.$

$Y = 1/8 x e^x + \cancel{3/8 x e^x - 3/32 e^x}$

$Y = 1/8 x e^x$



$2y_1'' = -4y_1 + 2(y_2 - y_1) \rightarrow 2y_1'' = -4y_1 + 2y_2 - 2y_1$

$1y_2'' = -2(y_2 - y_1) \rightarrow$

$2y_1'' + 6y_1 = 2y_2 \rightarrow \boxed{y_1'' + 3y_1 = y_2}$

$y_2'' = -2y_2 + 2y_1$

$\rightarrow (y_1'' + 3y_1)'' = -2y_1'' + 3y_1'' + 2y_1''$

$y_1^{(4)} + 3y_1'' = -2y_1'' - 4y_1$

$y^{(4)} + 5y'' + 4y = 0$

$r = \frac{q}{q}$ $\qquad r = (r^2+1)(r^2+4) = 0$

$r = \pm i \quad r = \pm 2i$

$y = C_1 \cos x + C_2 \sin x + C_3 \cos 2x + C_4 \sin 2x$