

5.4 Differentiation of Exponential Functions

By

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Let's consider the derivative of the exponential function.
Going back to our limit definition of the derivative:

$$\frac{d}{dx} [f(x)] = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx} (e^x) = \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h}$$

First rewrite the exponential using exponent rules.

$$= \lim_{h \rightarrow 0} \frac{e^x e^h - e^x}{h}$$

Next, factor out e^x .

$$= \lim_{h \rightarrow 0} \frac{e^x (e^h - 1)}{h}$$

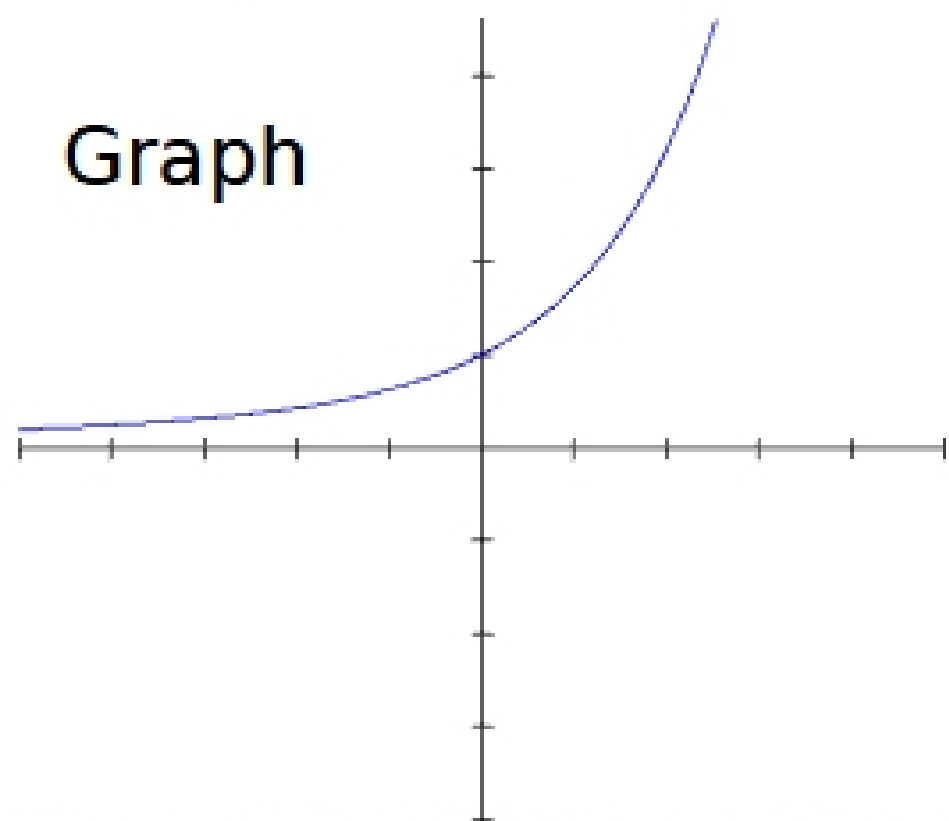
Since e^x does not contain h , we can move it outside the limit.

$$= e^x \lim_{h \rightarrow 0} \frac{e^h - 1}{h}$$

Substituting $h=0$ in the limit expression results in the indeterminate form $\frac{0}{0}$, thus we will need to determine it.

We can look at the graph of $f(x) = \frac{e^x - 1}{x}$ and observe what

happens as x gets close to 0. We can also create a table of values close to either side of 0 and see what number we are closing in on.



Table

x	-.1	-.01	-.001	.001	.01	.1
y	.95	.995	.999	1.0005	1.005	1.05

At $x = 0$, $f(0)$ appears to be 1. As x approaches 0, y approaches 1.