CHAPTER 9

# Factorial Experiments

## CONTENTS



### ■ Example 9.1: What Makes a Wiring Harness Last Longer?

Many electrical wiring harnesses, such as those used in automobiles and airplanes, are subject to considerable stress. Therefore, it is important to design such harnesses to prolong their useful life. The objective of this experiment is to investigate factors affecting the failure of an electrical wiring harness. The factors of the experiment are

STRANDS: the number of strands in the wire, levels are 7 and 9,
SLACK: length of unsoldered, uninsulated wire in 0.01 in., levels are 0, 3, 6, 9, and 12, and
GAGE: a reciprocal measure of the diameter of the wire, levels are 24, 22, and 20.

The response, CYCLES, is the number of stress cycles to failure, in 100 s.

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**Table 9.1** Cycles to Failure of a Wire Harness

| | Number of Strands | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Wire Slack** | **7** | | **7** | | **7** | | **9** | | **9** | | **9** | |
| **Wire Gage** | **24** | | **22** | | **20** | | **24** | | **22** | | **20** | |
| 0 | 2 | 4 | 14 | 9 | 6 | 8 | 3 | 3 | 10 | 14 | 12 | 11 |
| 3 | 5 | 2 | 6 | 15 | 5 | 7 | 2 | 5 | 17 | 17 | 16 | 8 |
| 6 | 6 | 3 | 14 | 7 | 6 | 5 | 5 | 5 | 10 | 10 | 10 | 8 |
| 9 | 9 | 16 | 12 | 12 | 8 | 12 | 6 | 4 | 16 | 11 | 13 | 7 |
| 12 | 14 | 12 | 10 | 14 | 12 | 11 | 13 | 15 | 20 | 17 | 12 | 15 |

*Note: Adapted from Enrick (1976).*

The experiment is a completely randomized design with two independent samples for each combination of levels of the three factors, that is, an experiment with a total of $2 \cdot 5 \cdot 3 = 30$ factor levels. The objective of the experiment is to see what combination of these factor levels maximizes the number of cycles to failure. The data are given in Table 9.1, which shows, for example, that 2 and 4 cycles to failure were reported for SLACK $= 0$, STRANDS $= 7$, and GAGE $= 24$ (*source:* Enrick, 1976). ■

## 9.1 INTRODUCTION

In Chapter 6 we presented the methodology for comparing means of populations that represent levels of a single factor. This methodology is based on a one-way or single-factor analysis of variance model. Many data sets, however, involve two or more factors. This chapter and Chapter 10 present models and procedures for the analysis of multifactor data sets. Such data sets arise from two types of situations:

1. *Factorial experiments*: In many experiments it is desirable to examine the effect of two or more factors on the same type of unit. For example, a crop yield experiment may be conducted to examine the differences in yields of several varieties as well as different levels of fertilizer application. In this experiment, variety is one factor and fertilizer is the other. An experiment that has each combination of all factor levels applied to the experimental units is called a **factorial experiment**. Although data exhibiting a multifactor structure arise most frequently from designed experiments, they may occur in other contexts. For example, data on test scores from a sample survey of students of different ethnic backgrounds from each of several universities may be considered a factorial "experiment," which can be used to ascertain differences on, say, mean test scores among schools and ethnic backgrounds.
2. *Experimental design*: It is often desirable to subdivide the experimental units into groups before assigning them to different factor levels. These groups are defined

in such a way as to reduce the estimate of variance used for inferences. This procedure is usually referred to as "blocking," and also results in multifactor data sets. Procedures for the analysis of data arising from experimental designs are presented in Chapter 10.

Actually, a data set may have both a factorial structure and include blocking factors. Such situations are also presented in Chapter 10.

As in the one-way analysis of variance, the analysis of any factorial experiment is the same whether we are considering a designed experiment or an observational study. The interpretation may, however, be different. Also, as in the one-way analysis of variance, the factors in a factorial experiment may have qualitative or quantitative factor levels that may suggest contrasts or trends, or in other cases may be defined in a manner requiring the use of post hoc paired comparisons.

## 9.2 CONCEPTS AND DEFINITIONS

In a factorial experiment we apply several factors simultaneously to each experimental unit, which we will again assume to be synonymous with an observational unit.

**Definition 9.1** *A **factorial experiment** is one in which responses are observed for every combination of factor levels.*

We assume (for now) that there are two or more independently sampled experimental units for each combination of factor levels and also that each factor level combination is applied to an equal number of experimental units, resulting in a **balanced** factorial experiment. We relax the assumption of multiple samples per combination in Section 9.6. Lack of balance in a factorial experiment does not alter the basic principles of the analysis of factorial experiments, but does require a different computational approach (see Chapter 11). A factorial experiment may require a large number of experimental units, especially if we have many factors with many levels. Alternatives are briefly noted in Section 9.7.

A classical illustration of a factorial experiment concerns a study of the crop yield response to fertilizer. The **factors** are the three major fertilizer ingredients: N (nitrogen), P (phosphorus), and K (potassium). The **levels** are the pounds per acre of each of the three ingredients, for example:

N at four levels: 0, 40, 80, and 120 lb. per acre,
P at three levels: 0, 80, and 160 lb. per acre, and
K at three levels: 0, 40, and 80 lb. per acre.

The **response** is yield, which is the variable to be analyzed.

The set of factor levels in the factorial experiment consists of all combinations of these levels, that is, $4 \times 3 \times 3 = 36$ combinations. In other words, there are 36