EE 428 **EXAM I** **2 October 1996**

Name: Solutions

ID#: _____

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Problem	Weight	Score
1	30	
2	35	
3	35	
Total	100	

This test consists of three problems. Answer each problem on the exam itself; if you use additional paper, repeat the identifying information above, and staple it to the rest of your exam when you hand it in. The quality of your analysis and evaluation is as important as your answers. Your reasoning must be precise and clear; your complete English sentences should convey what you are doing.

Problem 1: (30 points)

The depth y of a submarine can be varied by adjusting the angle θ of the stern plane.

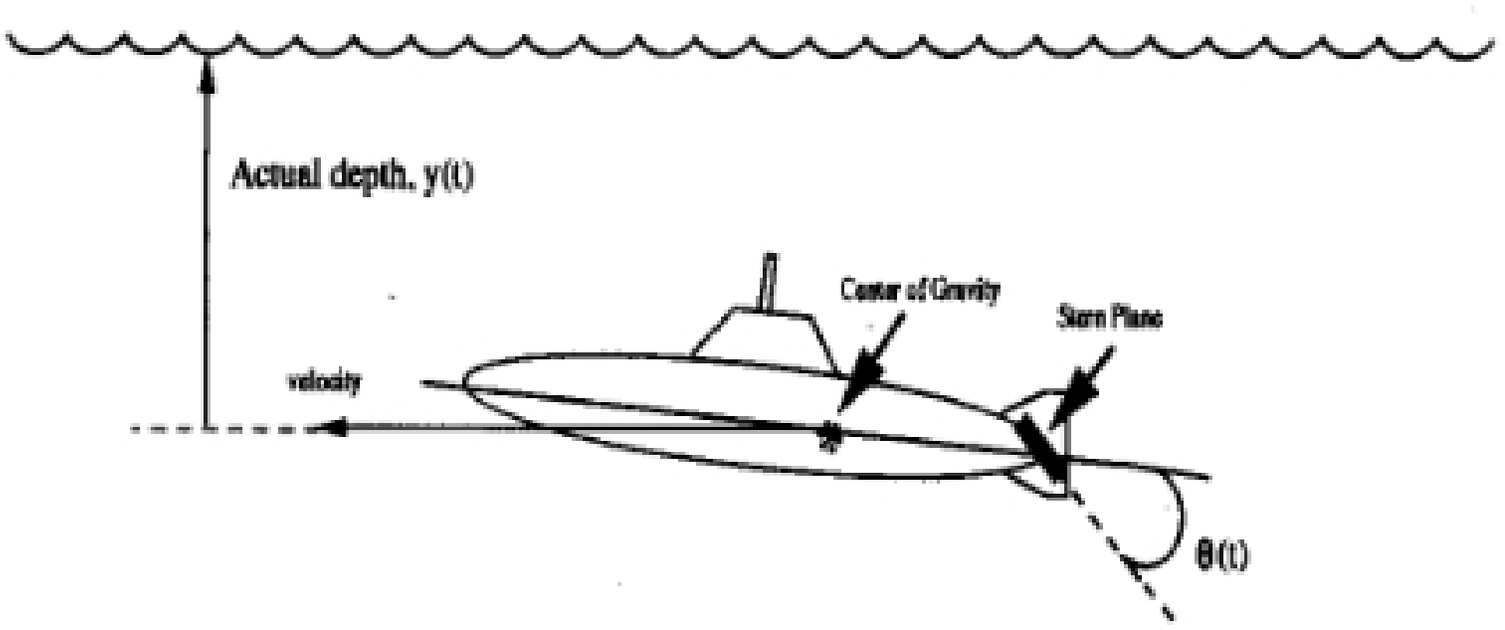


Figure 1: Sketch of the submarine system.

The relationship between the depth $y(t)$ and stern angle θ is described by the transfer function

$$G(s) = \frac{Y(s)}{\Theta(s)} = \frac{(s+1)^2}{s(s^2+0.09)}.$$

1. **(10 points) Represent the system using a single ordinary differential equation.**
2. **(10 points) Draw an all-integrator block diagram that describes the system dynamics.**
3. **(10 points) Obtain a state-space representation of the system.**

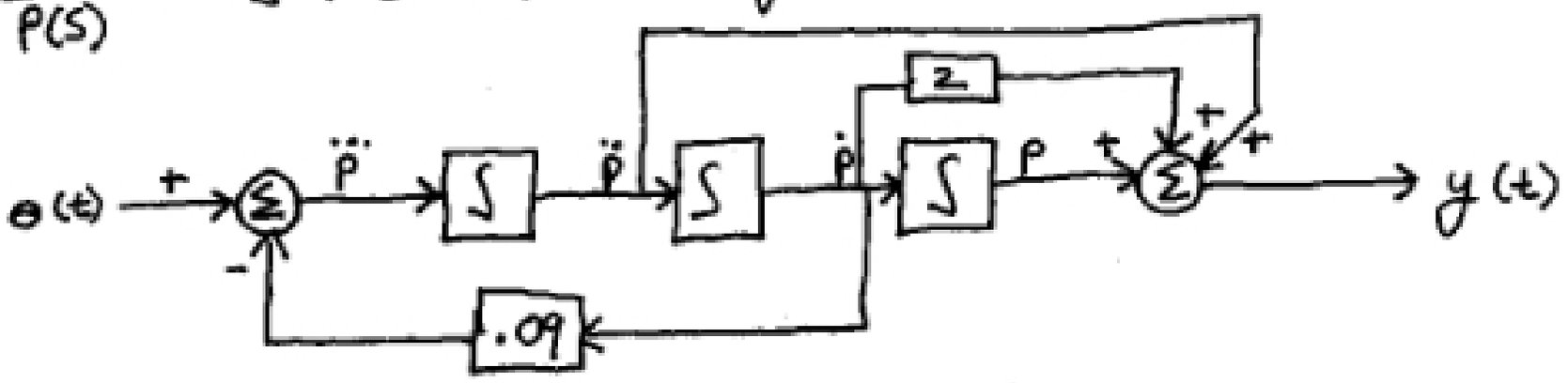
1. $Y(s)\{s^3 + 0.09s\} = \{s^2 + 2s + 1\}\Theta(s)$

$$\dddot{y}(t) + 0.09\,\dot{y}(t) = \ddot{\theta}(t) + 2\dot{\theta}(t) + \theta(t)$$

2. $\frac{Y(s)}{\Theta(s)} = \frac{P(s)}{\Theta(s)}\,\frac{Y(s)}{P(s)}$

$$\frac{P(s)}{\Theta(s)} = \frac{1}{s^3+0.09s} \longleftrightarrow \dddot{p} + 0.09\dot{p} = \theta(t)$$

$$\frac{Y(s)}{P(s)} = s^2 + 2s + 1 \longleftrightarrow y(t) = \ddot{p}(t) + 2\dot{p}(t) + p(t)$$



3 Let

$$\left.\begin{aligned} x_1 &= p \\ x_2 &= \dot{p} \\ x_3 &= \ddot{p} \end{aligned}\right\}$$

state variables assigned to integrator outputs. Other methods of choosing state variables are also available.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -.09\,x_2 + \cdot\,\theta(t)$$
$$y(t) = x_1 + 2x_2 + x_3$$

In matrix form,

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -0.09 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \theta(t)$$

$$y(t) = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$